

# MT 2008 First Year Relativity - Problems 1

## Special Relativity: Dr R A Taylor

The problems have been drawn from various sources. I suggest that the problems marked with a † are **all** worth doing. Attempt a selection of the remaining problems from each section. The problems marked with \* are somewhat harder. Answers are given to some of the problems, but they are not guaranteed.

### SECTION A – Lorentz Transformation and Elementary Consequences

1. The space and time coordinates of two events as measured in a frame S are as follows:

$$\text{Event 1: } x_1 = x_0, \quad t_1 = x_0/c \quad (y_1 = 0, \quad z_1 = 0)$$

$$\text{Event 2: } x_2 = 2x_0, \quad t_2 = x_0/2c \quad (y_2 = 0, \quad z_2 = 0)$$

Show that there exists an inertial frame in which these events occur at the same time, and find the velocity of this frame relative to S. At what value of time in the new frame do both events occur?

$$[\text{Ans: } \beta = -1/2, \quad t' = \sqrt{3}x_0/c.]$$

2. A small, powerful laser is placed on a turntable that rotates at 1200 rev/s. The laser, whose beam makes a  $30^\circ$  angle with the horizontal, shines on clouds 50 km away. Calculate the speed with which the light spot on the clouds moves. Does this speed violate the limitation of the speed of light? Explain.

$$[\text{Ans: } 3.3 \times 10^8 \text{ ms}^{-1}]$$

\*3. Two observers *A* and *B*, in inertial frames, have identical clocks. *B* passes close to *A* with a relative velocity  $0.5c$ .  $t$  seconds after *B* has passed, *A* flashes him a light signal.  $2t$  seconds after receiving this signal (as measured on his own clock), *B* acknowledges by a return light signal to *A*. When does *A* receive it?

$$[\text{Ans: } (3 + 2\sqrt{3})t.]$$

†4. Proxima Centauri, the star nearest our own, is some 4.2 ly away. (a) If a spaceship could travel at a speed of  $0.80c$ , how long would it take to reach the star, according to the spaceship's pilot? (b) What would someone in the frame that moves with the spaceship measure as the distance to Proxima Centauri?

$$[\text{Ans: (a) 3.1 yr, (b) 2.5 ly}]$$

5. A new Klingon battleship races at a top speed of  $0.2c$  away from the planet XG4C. The starship *Enterprise* follows at a speed of  $0.25c$  relative to the Klingon ship. With what speed does the *Enterprise* appear to catch up with the Klingon "wessel", according to Mr. Chekov on the planet? [Ans:  $0.23c$ ]

†6. The Galaxy is about  $10^5$  light years across and the most energetic cosmic rays known have energies of the order of  $10^{19}$  eV. How long would it take a proton with this energy to cross the Galaxy as measured in the rest frame of (i) the Galaxy and (ii) the proton. The rest-mass of the proton is  $938 \text{ MeV}/c^2$ .

$$[\text{Ans: (i) } 10^5 \text{ years, (ii) 4.96 min.}]$$

†7. The proper mean lifetime of  $\pi^+$  mesons is 26 ns.

(a) What is the mean lifetime of a burst of  $\pi^+$  mesons travelling with  $\beta = 0.73$ ?

(b) What distance is travelled at this velocity during one mean lifetime?

(c) What distance would be travelled without the effect of time dilation?

[Ans: (a) 38 ns, (b) 8.32 m, (c) 5.69 m.]

8. A beam of unstable  $K^+$  mesons, travelling at a speed of  $0.866c$  passes through two counters 9m apart. The particles suffer a negligible loss of speed and energy in passing through the counters but give signals that can be counted. It is observed that 1000 counts are recorded in the first counter and 246 in the second. Assuming that the whole of this decrease is due to the decay of the particles in flight, what is their half-life in their own rest frame.

[Ans: 8.6 ns.]

†9. A burst of muons is produced by a cosmic ray interacting in the upper atmosphere. They travel towards the Earth's surface with an average speed of  $0.99c$ . If 1% of the burst survive to reach ground level, estimate the height of the burst and calculate the distance as measured in the muon's frame. The muon mean lifetime is  $2.2 \mu\text{sec}$ .

[Ans:  $2.13 \times 10^4$  m,  $3.01 \times 10^3$  m.]

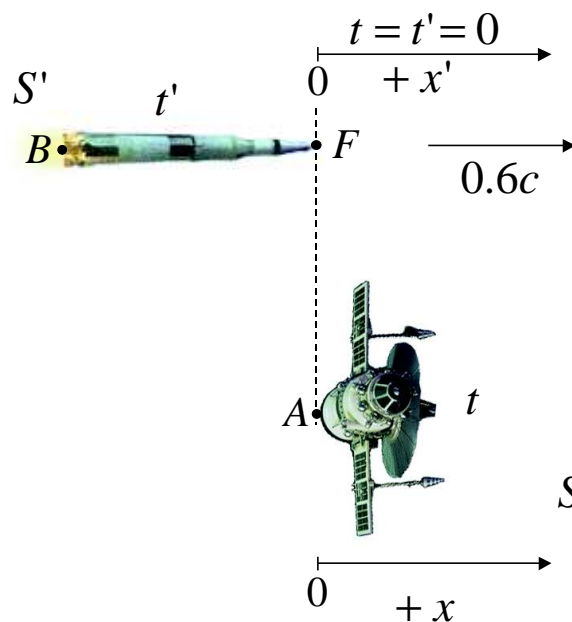
\*10. Twin paradox for fun - C Darwin Nature **180** 976 (1957). On New Year's day 2030 an astronaut (A) sets out from Earth at speed  $0.8c$  to travel to the nearest star  $\alpha$ -Centauri, about 4 light-years away (as measured in the Earth frame of reference). On reaching the star A immediately turns around and returns to Earth arriving back on New Year's day 2040 (Earth time). A has a sibling B who remains on Earth and they agree to send each other greetings by radio every New Year's day until A returns.

(a) Satisfy yourself that A sends only 6 messages (including the one on the last day of the trip, whereas B sends 10.

(b) Draw a space-time diagram of A's journey in the Earth frame (mark the scales of  $x$  and  $ct$  in light years). Draw the world lines of all the radio signals that B transmits. Use the diagram to verify that A (astronaut) has received only 1 signal up to the moment of reaching the star and turning back, and receives the other 9 during the return half of the trip.

(c) On a second space-time diagram (again in the Earth frame) show the world lines of the astronaut (A) and all the signals that A sends. Verify that B (on Earth) receives one message each 3 years of Earth time for the first 9 years of the journey, then receives 3 more messages in the last year to make the total of 6 - which is the expected total since according to A's time the trip takes 3 years out and 3 years back.

\*11. A spaceship of length 30 m travels at  $0.6c$  past a satellite. Clocks in frame  $S'$  of the spaceship and  $S$  of the satellite are synchronized within their respective frames of reference and are set to zero so that  $t = t' = 0$  at the instant the front of the spaceship  $F$  passes point  $A$  on the satellite, located at  $x = x' = 0$  (see diagram below). At this time a light flashes at  $F$ . (a) What is the length of the ship as measured by an observer on the satellite? (b) What time does the observer on the satellite read from her clock when the trailing edge  $B$  of the spaceship passes her? (c) When the light flash reaches  $B$  at the rear of the spaceship, what is the reading  $t_1'$  of a clock at  $B$ ? (d) What is reading  $t_1$  on the clock on the satellite when, according to the observer on the satellite, the flash reaches  $B$ ? [Ans: (a) 24 m, (b)  $13.3 \times 10^{-8}$  s, (c)  $10 \times 10^{-8}$  s, (d)  $5.0 \times 10^{-8}$  s]



### SECTION B – Addition of velocities, Doppler shift

†12. Show that two successive Lorentz transformations with velocities  $v_1$  and  $v_2$  in the same direction are equivalent to a single Lorentz transformation with velocity  $(v_1 + v_2)/(1 + v_1 v_2/c^2)$ .

†13. A new Klingon battleship has a proper length of 217 m and travels at a speed of  $0.20c$  with respect to its home planet. The Klingons prepare to battle the *Enterprise*, which is moving at the same speed with respect to the same planet. If the Klingons are heading straight at the *Enterprise*, what is the length of the Klingon ship as measured by Captain Kirk?

[Ans: 200 m]

†14. A driver was caught running a red light. His defence is that he saw the light as green, as a result of the Doppler shift. He is arrested. What for? Estimate the seriousness of his transgression.

†15. By 2025 a spaceship shuttle service operates between the Earth and Mars. Each spaceship is equipped with identical monochromatic head and tail lights, and their cruising speed  $v$  relative to earth, is such that the headlight of a homeward-bound spaceship appears green ( $\lambda = 500 \text{ nm}$ ) and the tail-light of a departing spaceship appears red ( $\lambda = 600 \text{ nm}$ ).

(i) What is  $v/c$ ?

(ii) What is the wavelength of the headlight of an earth-bound spaceship is observed from an outward bound one?

[Ans: (i)  $1/11$ , (ii)  $457 \text{ nm}$ .]

\*16. Light of frequency  $\nu_0$  is emitted from a source moving away from an observer with speed  $\beta c$ . If the light is emitted along the line of the motion show that the frequency measured by the observer is  $\nu = \nu_0 \gamma (1 - \beta)$ . An absorption line in  $\text{Ca}^+$  has a wavelength of  $394 \text{ nm}$  as measured for a stationary source. The same line in the spectrum of Hydra occurs at  $475 \text{ nm}$ . What can be deduced about the motion of Hydra with respect to the Earth?