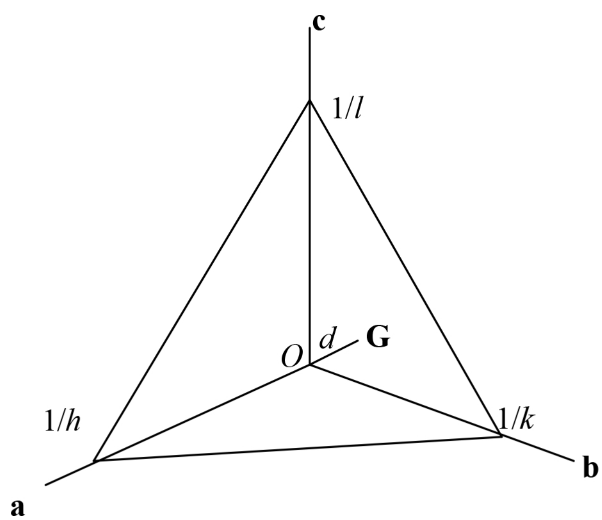
Inter-plane spacing



If $\mathbf{G}$ is perpendicular to 2 lines in the plane, then $\mathbf{G}$ is perpendicular to the plane. So, with

$$\mathbf{G} = h\mathbf{A} + k\mathbf{B} + l\mathbf{C} \quad \& \quad \mathbf{A} = \frac{2\pi\mathbf{b} \wedge \mathbf{c}}{\mathbf{a}.(\mathbf{b} \wedge \mathbf{c})}$$

consider any vector in the plane and take the dot product

$$\left(\frac{1}{k}\mathbf{b} - \frac{1}{h}\mathbf{a}\right).(h\mathbf{A} + k\mathbf{B} + l\mathbf{C}) = 2\pi - 2\pi = 0$$

by the definition of reciprocal lattice vectors. Similarly for another line in the plane the same holds. Therefore $\mathbf{G}$ is perpendicular to the plane and d is along $\mathbf{G}$.

So now project $\frac{1}{k}\mathbf{b}$ along $\frac{\mathbf{G}}{|\mathbf{G}|}$ (unit vector) to get:

$$d = \frac{1}{k}\mathbf{b}.\frac{\mathbf{G}}{|\mathbf{G}|} = \frac{2\pi}{|\mathbf{G}|}$$

We also have, therefore

$$d^2 = \frac{4\pi^2}{\mathbf{G}.\mathbf{G}}$$

but for a cubic lattice, $\mathbf{A} = \frac{2\pi}{a}\hat{\mathbf{x}}$ etc., where $\mathbf{a} = a\hat{\mathbf{x}}$ and so on. Therefore

$$|\mathbf{G}| = \frac{2\pi}{a}(h^2 + k^2 + l^2)^{\frac{1}{2}}$$

so

$$d^2 = \frac{a^2}{(h^2 + k^2 + l^2)} \quad \& \quad d = \frac{a}{(h^2 + k^2 + l^2)^{\frac{1}{2}}}$$