Energy & Momentum Summary

Kinetic energy W is

$$W = mc^2(\gamma - 1)$$

When $v = 0$, $\gamma = 1$, $W = 0$

Define Energy

$$E = W + mc^2$$

$$E = \gamma mc^2$$

$$E^2 - p^2c^2 = m^2c^4$$ **INVARIANT**

or

$$p^2c^2 - E^2 = -m^2c^4$$

Now when $\gamma = 1$, $E = mc^2$

m = **REST MASS** of object.

$$x^2 - c^2t^2 = -c^2\tau^2$$ **INVARIANT**

Transformation of *E* and *p*

$$p'_x = \gamma(p_x - \beta E/c)$$

$$p'_y = p_y$$

$$p'_z = p_z$$

$$E'/c = \gamma(E/c - \beta p_x)$$

Therefore once more we may define a 4-vector such that:

$$X_\mu = L_{\mu\nu} X_\nu$$

where X is a 4-vector & $L_{\mu\nu}$ is the Lorentz Transformation matrix.

$$x_\mu = (x, y, z, ict)$$

$$p_\mu = (p_x, p_y, p_z, iE/c)$$ and $$L_{\mu\nu} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$k_\mu = (k_x, k_y, k_z, i\omega/c)$$

Relativistic Kinematics

Particle physics units:

m = MeV/c^2 p = MeV/c E = MeV 1 eV = 1.6 x 10^{-19} J

Then

$$E^2 = p^2 + m^2$$

In particle physics $\beta \approx 1$ therefore

$$E = \gamma m c^2 \quad \& \quad |\mathbf{p}| = \gamma m c \beta$$

thus

$$\gamma = E/mc^2 \quad \& \quad \beta = |\mathbf{p}|c/E$$

Therefore in particle physics units:

$$\gamma = E/m \quad \& \quad \beta = |\mathbf{p}|/E$$

and

$$t_{\mathrm{part}} = \gamma \tau_{\mathrm{part}} = \frac{E \tau_{\mathrm{part}}}{m}$$

Centre of Mass or Centre of Momentum

$$S = \left[\sum_i E_i\right]^2 - \left[\sum_i \mathbf{p}_i c\right]^2$$

Is **INVARIANT** for a group of particles.

In C of M frame:

$$S = \left[\sum_i E_i^*\right]^2 = E_{\rm cm}^2$$

Where E_i^* is the energy of the $i^{\rm th}$ particle in that frame.

Then:

$$\gamma_{\rm cm} = \frac{\sum_i E_i}{E_{\rm cm}} \quad \& \quad \beta_{\rm cm} = \frac{|\sum_i \mathbf{p}_i c|}{\sum_i E_i}$$

Threshold Energies

$$\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + (\bar{\mathrm{p}} + \mathrm{p}) \qquad E_{\mathrm{cm}} = 4 m_{\mathrm{p}} c^2$$

$$S = E_{\mathrm{cm}}^2$$ is invariant

Lab Frame:

a $\qquad$ b

$E_a^L, \mathbf{p}$ $\qquad$ $m_{\mathrm{p}} c^2$

$$S = 2 m_{\mathrm{p}} c^2 (m_{\mathrm{p}} c^2 + E_a^L)$$

$$\boxed{E_a^L = 7 m_{\mathrm{p}} c^2}$$

N.B. Lorentz transformations are not needed. In general:

$$\boxed{E_{\mathrm{cm}} = (m_1^2 + m_2^2 + 2 m_2 E_1)^{\frac{1}{2}}}$$

CERN – on the Swiss/French border

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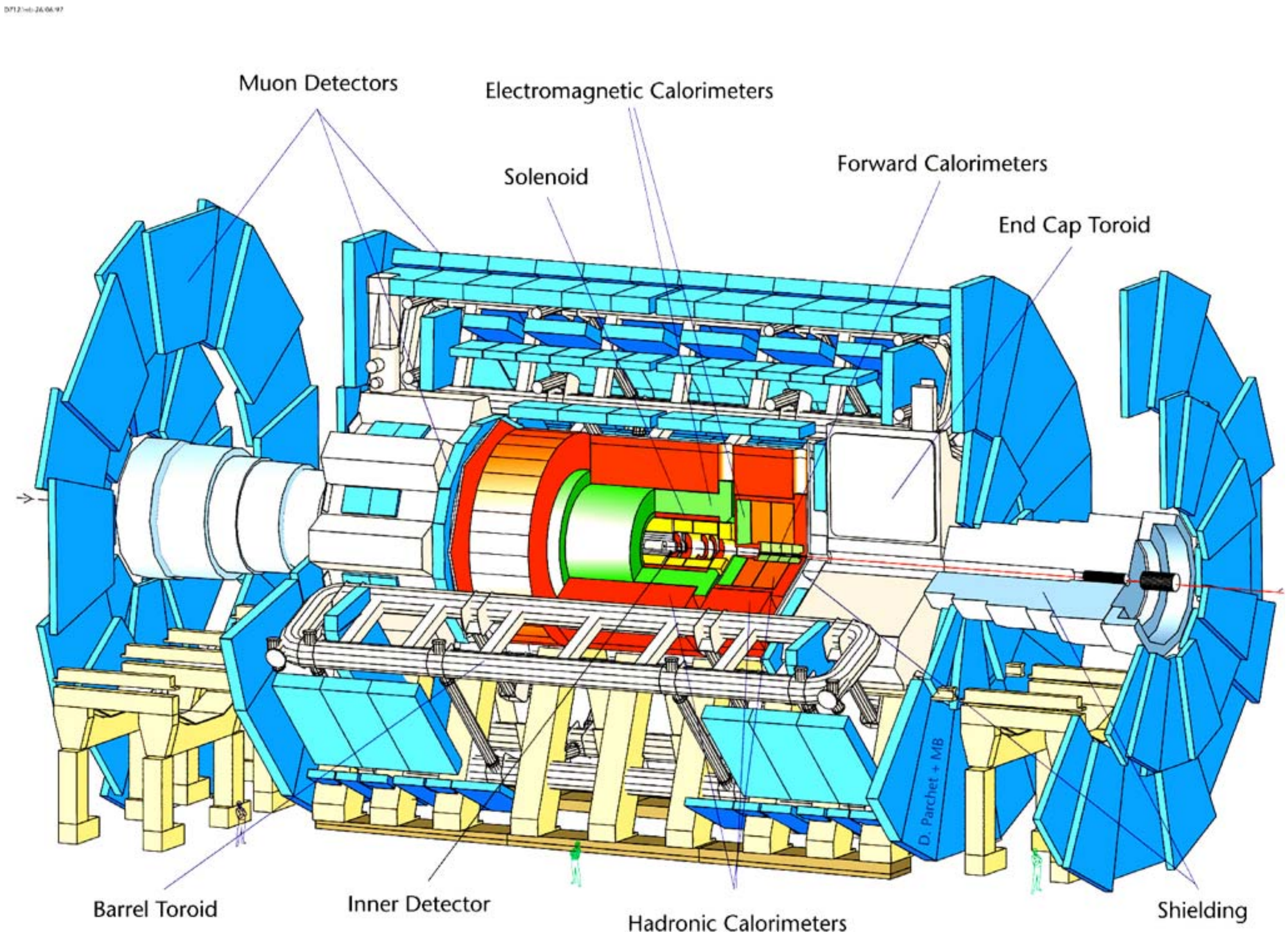
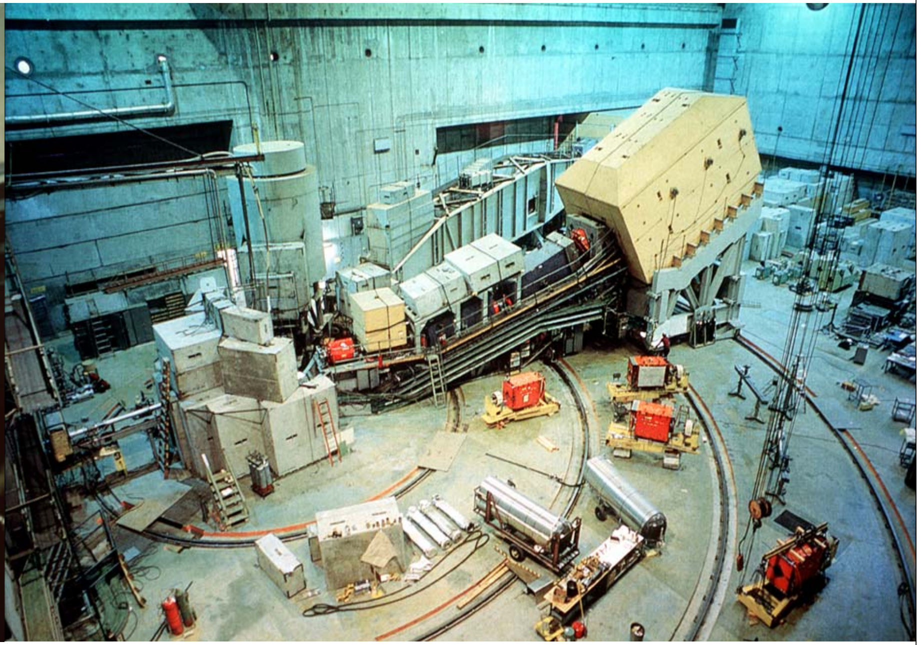
The ATLAS detector at CERN's LHC collider



ATLAS detector at the **Large Hadron Collider** (LHC) at CERN (which will accelerate each of two counter-rotating beams of protons to 7 TeV per proton). The detector will look for the Higgs boson.

DESY – in Hamburg, Germany



The accelerator and tunnel at LEP, CERN before it was ripped out for the LHC which will switch on in late 2007.

Particle detector at DESY in Hamburg

The Stanford Linear Accelerator (SLAC) in California