# What About Momentum?

Newton says:

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = m\mathbf{a}$$

We also have:

**Conservation of momentum**

**Conservation of energy**

---

As $\mathbf{p} \to \infty$, classically $\mathbf{v} \to \infty$

This is **not** allowed in special relativity.

Therefore: $\mathbf{p}_{\mathrm{rel}} = m_0 f(v)\mathbf{v}$

In fact:

$$\boxed{\mathbf{p}_{\mathrm{rel}} = \gamma m_0 \mathbf{v}}$$

Where $m_0$ is the **REST MASS.**

## Newton's Second Law

$$\mathbf{F}_{\mathrm{rel}} = \frac{\mathrm{d}\mathbf{p}_{\mathrm{rel}}}{\mathrm{d}t} = m\frac{\mathrm{d}}{\mathrm{d}t}\left[\gamma\mathbf{v}\right]$$

$$\mathbf{F}_{\mathrm{rel}} = \gamma m\mathbf{a} + m\mathbf{v}\frac{\mathrm{d}\gamma}{\mathrm{d}t}$$

Therefore Force = mass x acceleration is **not** valid in special relativity.

## Lorentz Force

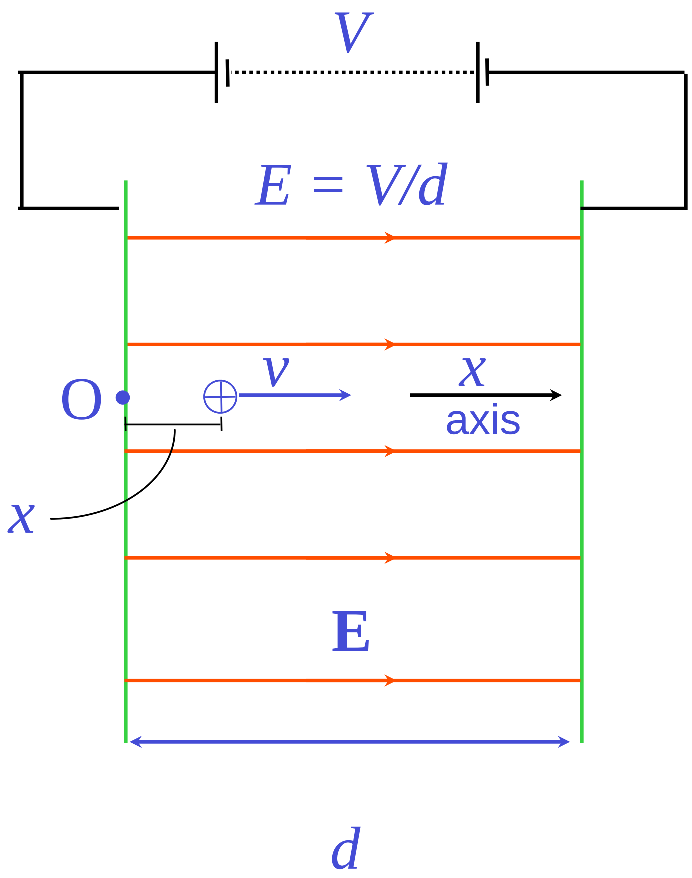
$$\mathbf{F} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\gamma m\mathbf{v}\right) = q\mathbf{E} + q\mathbf{v}\wedge\mathbf{B}$$

$q$ = charge (INVARIANT)

$\mathbf{E}$ = electric field

$\mathbf{B}$ = magnetic flux density

# Capacitor Example




$$\frac{\mathrm{d}}{\mathrm{d}t} = \left[ \frac{v}{(1 - v^2/c^2)^{\frac{1}{2}}} \right] = \frac{qE}{m}$$

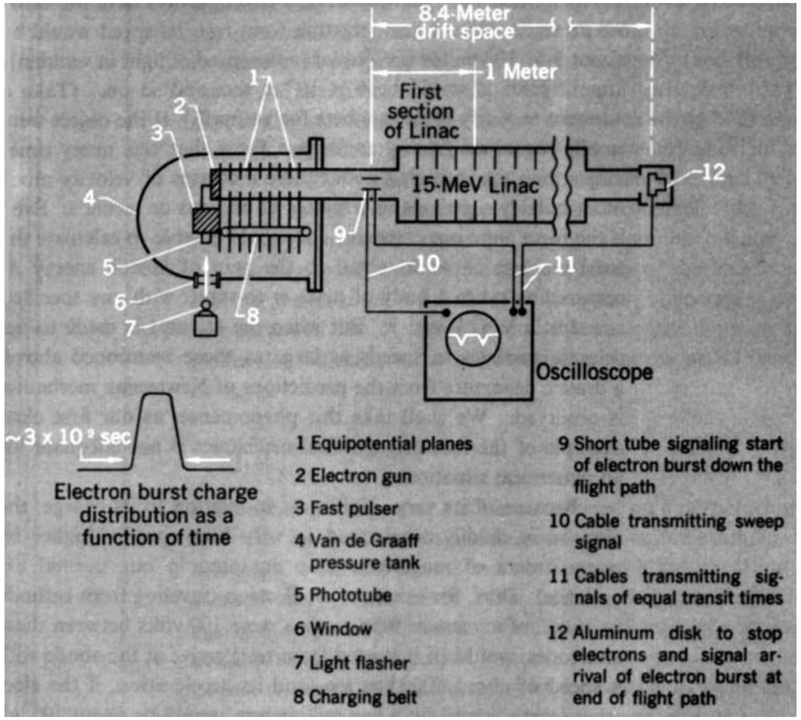
$$v = \frac{c}{(1 + m^2c^2/q^2E^2t^2)^{\frac{1}{2}}}$$

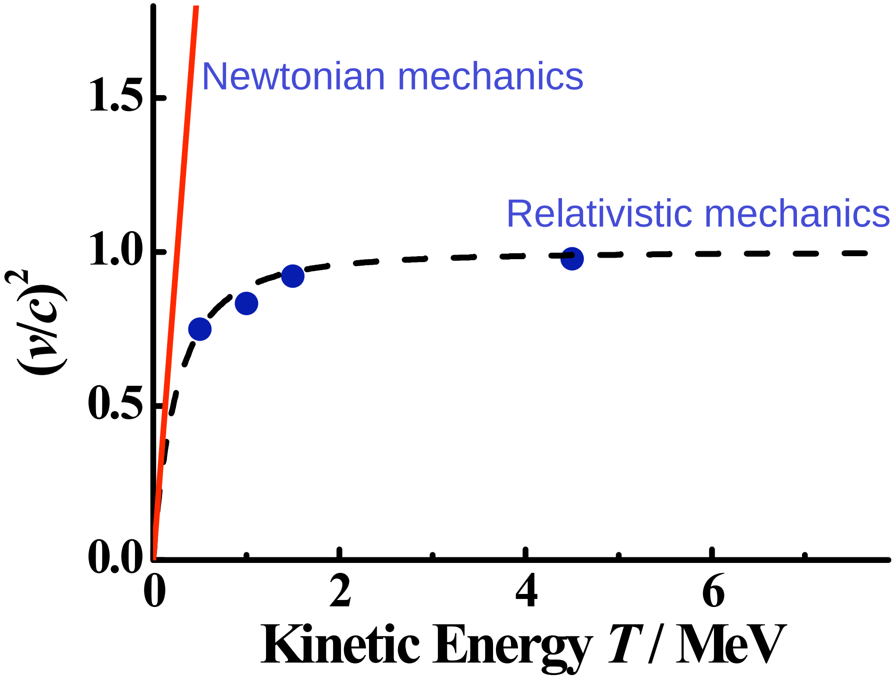
for $v << c$

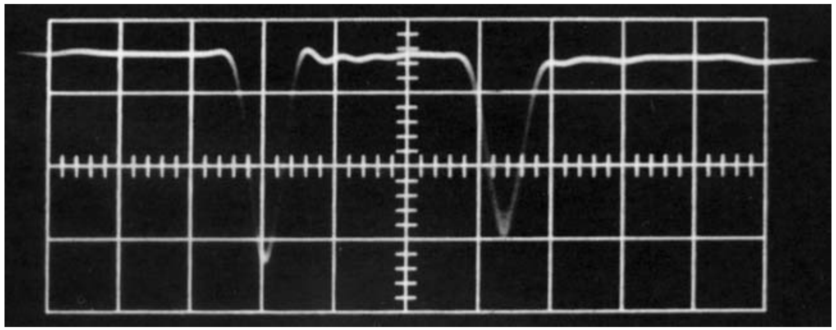
$$v \approx \frac{qEt}{m}$$

(as required by Newton)

# Momenergy



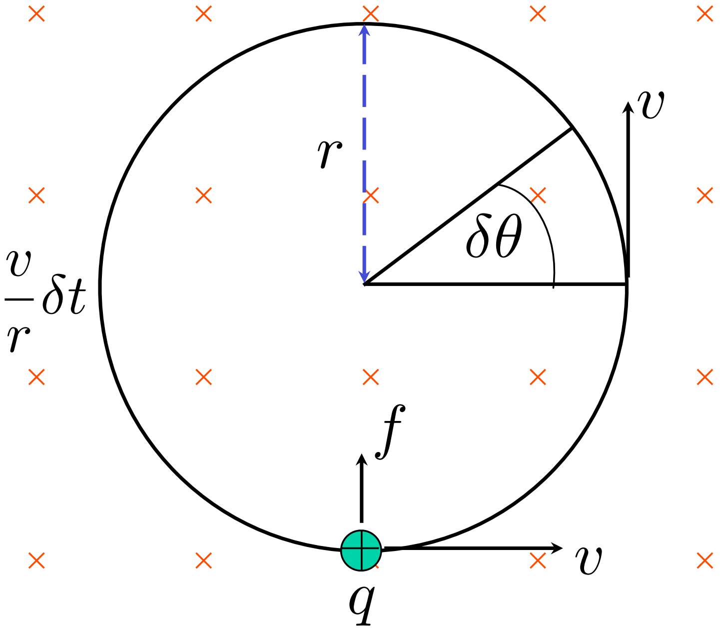



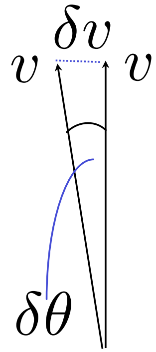
1ns/division

# Motion in a Magnetic Field

**B** points down into page




$$\delta\theta = \omega\delta t = \frac{v}{r}\delta t$$




$$\frac{\delta v}{v} \approx \delta\theta$$

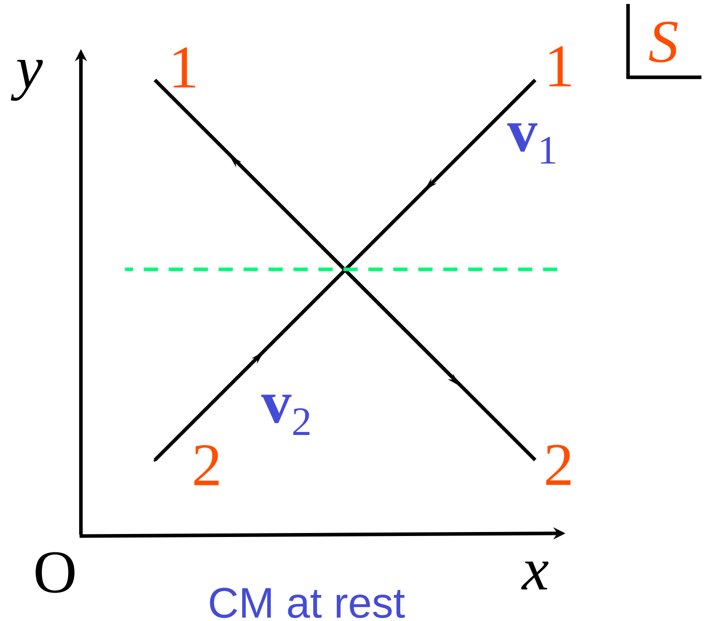
$$\approx \frac{v}{r}\delta t$$

$$a = \frac{\delta v}{\delta t} = \frac{v^2}{r}$$

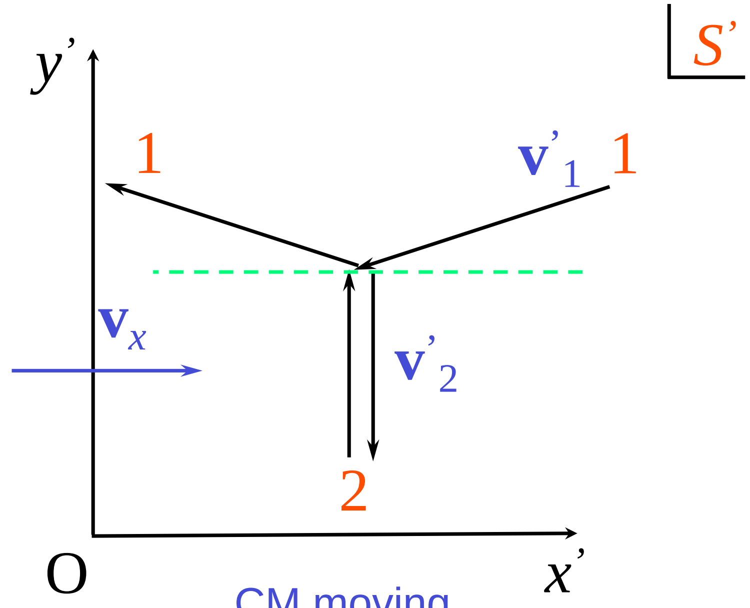
$$f_{\text{mag}} = qvB = \gamma m \frac{\mathrm{d}v}{\mathrm{d}t}$$

$$a = \frac{qvB}{\gamma m} = \frac{v^2}{r} \quad \& \quad r = \frac{\gamma m v}{qB} \quad \& \quad \omega = \frac{qB}{\gamma m}$$

# Elastic Collision




CM at rest




CM moving

| | $\mathbf{v}_1$ | $\mathbf{v}_2$ | | $\mathbf{v}'_1$ | $\mathbf{v}'_2$ |
|---|---|---|---|---|---|
| | $(-v_x, -v_y)$ | $(v_x, v_y)$ | **Before** | $(-v'_{x'}, -v'_{1y'})$ | $(0, v'_{2y'})$ |
| | $(-v_x, v_y)$ | $(v_x, -v_y)$ | **After** | $(-v'_{x'}, v'_{1y'})$ | $(0, -v'_{2y'})$ |

$\sum mv = 0$ in y direction

$\sum mv' \neq 0$ in y' direction since

$$v'_{1y'} \neq v'_{2y'}$$

# Relativistic Energy

$$W = \int F \mathrm{d}x = \int_0^t F v \mathrm{d}t$$

Then:

$$Fv = mv^2 \frac{\mathrm{d}\gamma}{\mathrm{d}t} + m\gamma v \frac{\mathrm{d}v}{\mathrm{d}t}$$

$$W = mc^2(\gamma - 1) = mc^2 \left[ \frac{1}{(1 - v^2/c^2)^{\frac{1}{2}}} - 1 \right]$$

This is the **Relativistic Kinetic Energy**.

$$E = W + mc^2 = \gamma mc^2$$

Then

Thus $E^2 - p^2c^2$ is an **INVARIANT**

$$E^2 - p^2c^2 = m^2c^4$$

# And so we have :-