TIME DILATION

In the Lorentz Transformations the interval between 2 events is INVARIANT.

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2 \Delta t'^2$$

Now for 2 events in S' at the same place (e.g.clock ticks), we have $\Delta x' = 0$. This is the clock rest frame and the time interval between 2 such events is the proper time denoted by $\Delta\tau$. So:

$$c^2 \Delta\tau^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

Divide by Δt^2 then:

$$\left(\frac{\Delta\tau}{\Delta t}\right)^2 = 1 - v^2 / c^2 \qquad \left(\left.\frac{d\mathbf{x}}{dt}\right|_{S'} = \mathbf{v}\right)$$

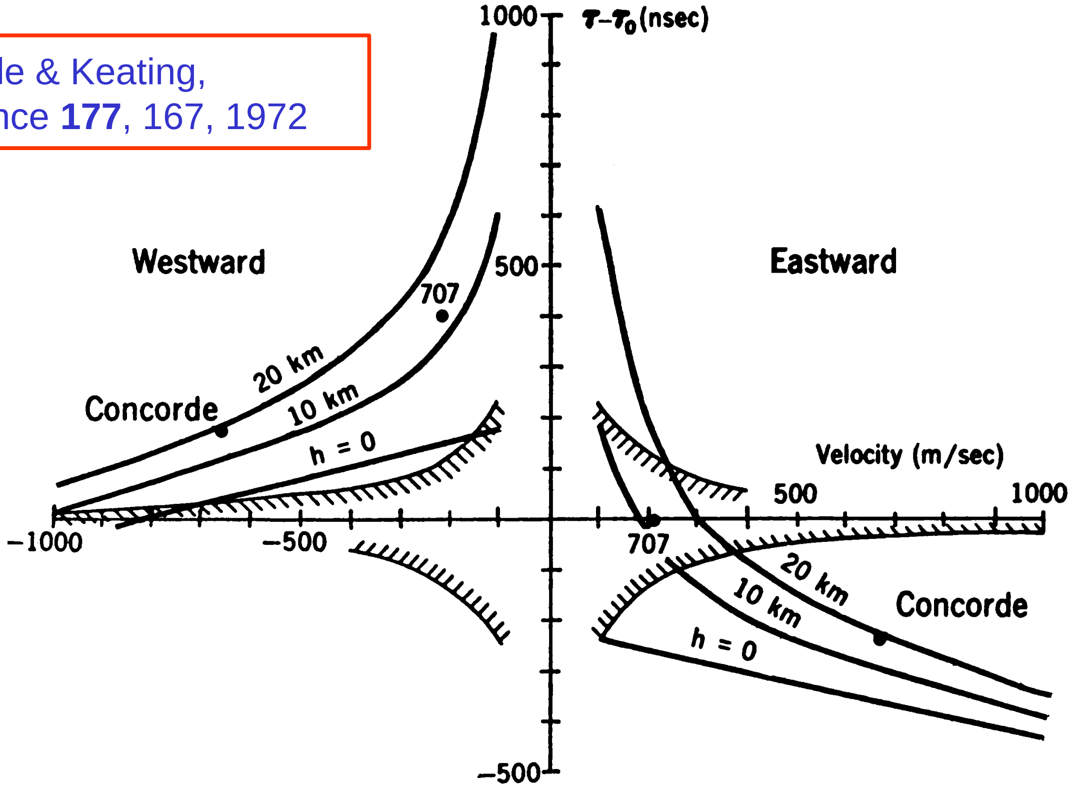
$$\therefore \Delta\tau = \Delta t \sqrt{1 - v^2 / c^2}$$

$$\therefore \Delta\tau < \Delta t$$

Therefore the time interval is longer than that in the rest frame - TIME DILATION

Around the World with Atomic Clocks

Hafele & Keating,
Science **177**, 167, 1972



Predicted relativistic time gain for a flying clock after a non-stop equatorial circumnavigation of the earth at various altitudes. The area within the hatched lines is below detection thresholds with a portable Cs clock.

$$t - t_0 = \left[\frac{gh}{c^2} - \left(2R\Omega v + v^2 \right) / 2c^2 \right] t_0$$

Gravitational "red shift"   Earth's rotation   Time dilation

Where τ_0 is clock at rest on Earth; v is the ground speed of the aircraft

Results (ns)	East	West	
	144 ± 14	179 ± 18	Gravity
Prediction	-184 ± 18	96 ± 10	Rotation/ Kinematic
	-40 ± 23	275 ± 21	Net
Measurement	-59 ± 10	273 ± 7	

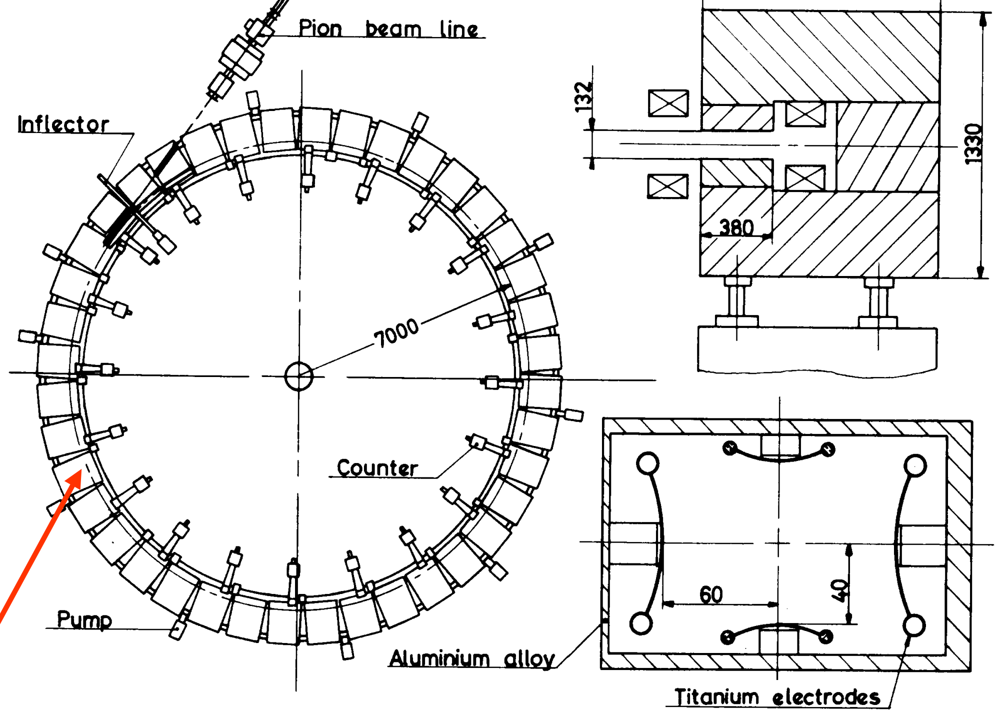
Muon Storage Ring Experiment – CERN 1977

3.1 GeV/c π⁻ beam

$$p^- \to m^- + \bar{n}_m$$

From PS accelerator

Magnet

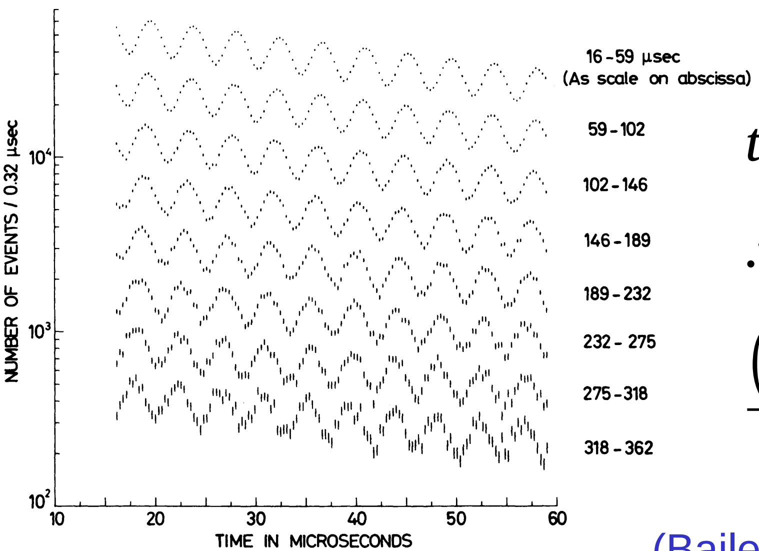


Muons circulate inside vacuum pipe, momentum = 3.098 GeV/c
γ = 29.33, β = 0.994

Electric quadrupole focusing unit

Decay: $m^- \to e^- + \bar{n}_e + n_m$ $\qquad t_0 = 2.2\ ms$

Muon Decay Spectrum in Flight



$$t_f = 64.368(29)\ ms$$

$$\therefore t_0 = 2.1948(10)\ ms$$

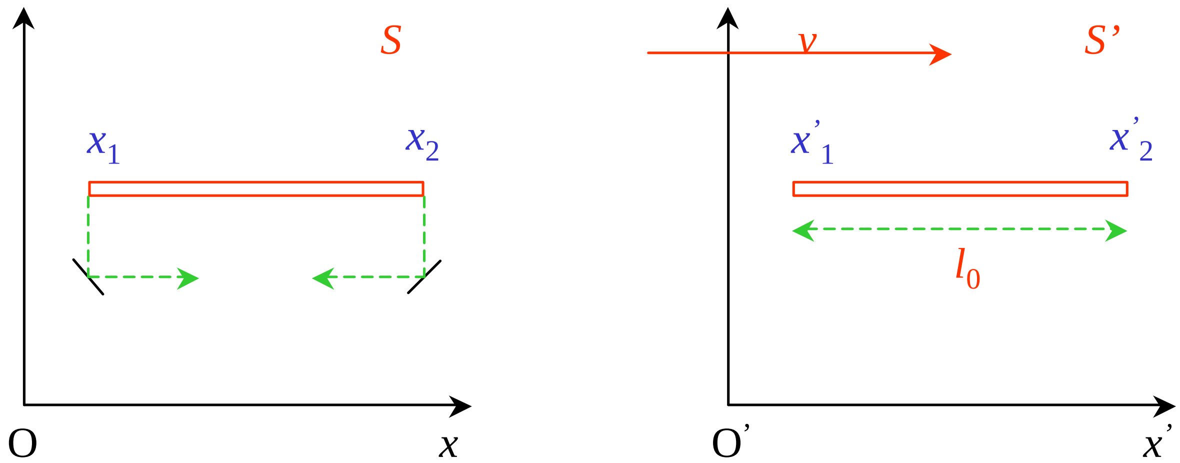
$$\frac{(t_0 - t_f/g)}{t_0} = (2 \pm 9) \times 10^{-4}$$

(Bailey et al, Nature **268**, 301, 1977)

LORENTZ CONTRACTION

Should be called Lorentz-Fitzgerald contraction

Consider a rigid rod of length l_0 at rest in frame S', moving with velocity v with respect to S.



In S' use a metre rule to measure l_0.

$$\Delta x' = x_2' - x_1' = l_0$$

In S , use light signals as shown to measure x_1 and x_2 at the SAME TIME

From the Lorentz transformations,

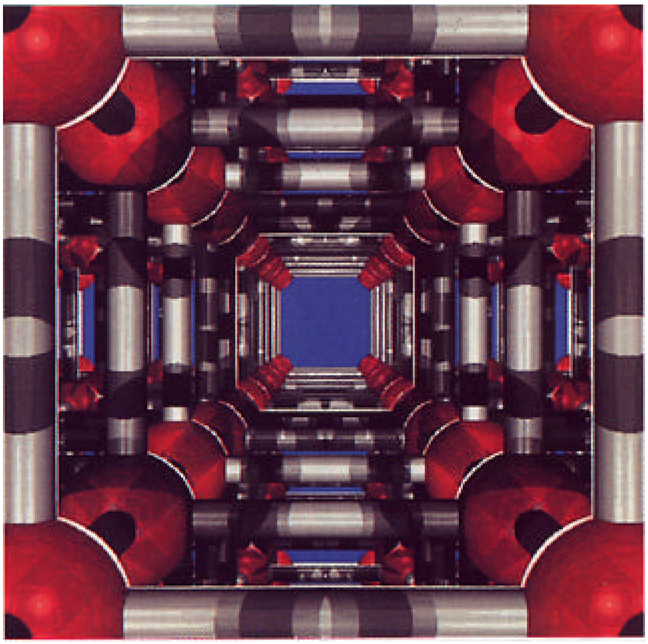
$$x_1' = \gamma(x_1 - vt)$$

$$x_2' = \gamma(x_2 - vt)$$

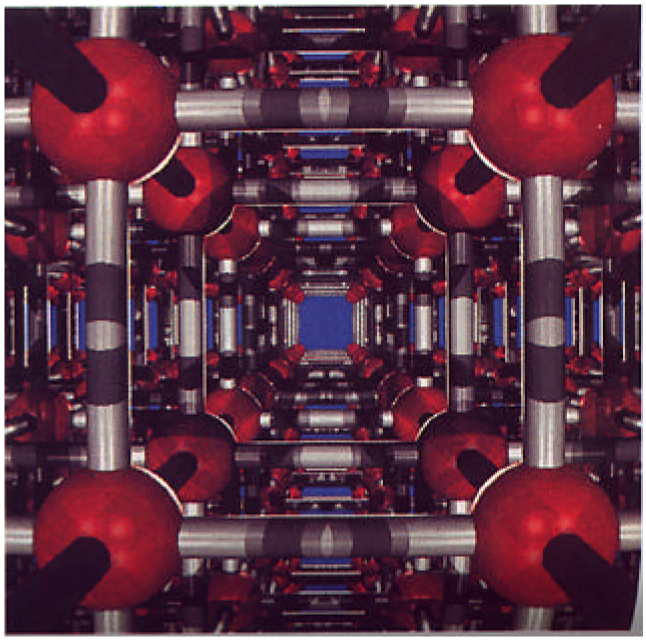
$$\Delta x' = \gamma \Delta x$$

$$\Delta x = l_0 / \gamma$$

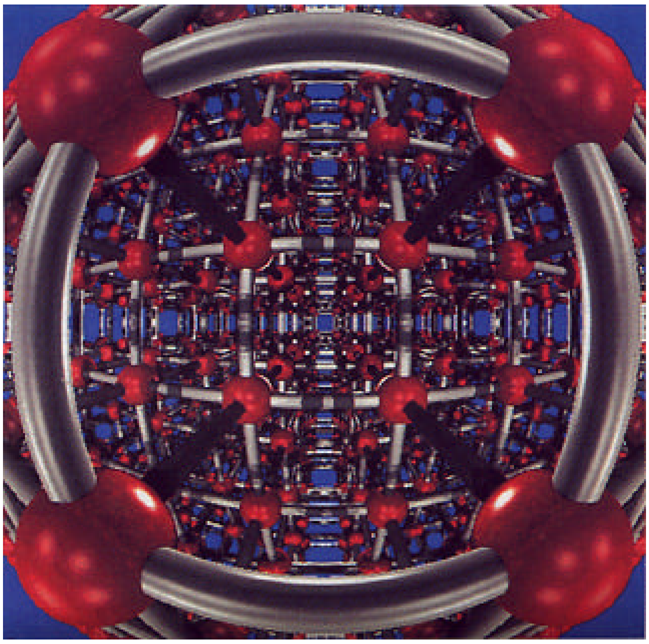
What Would You See?



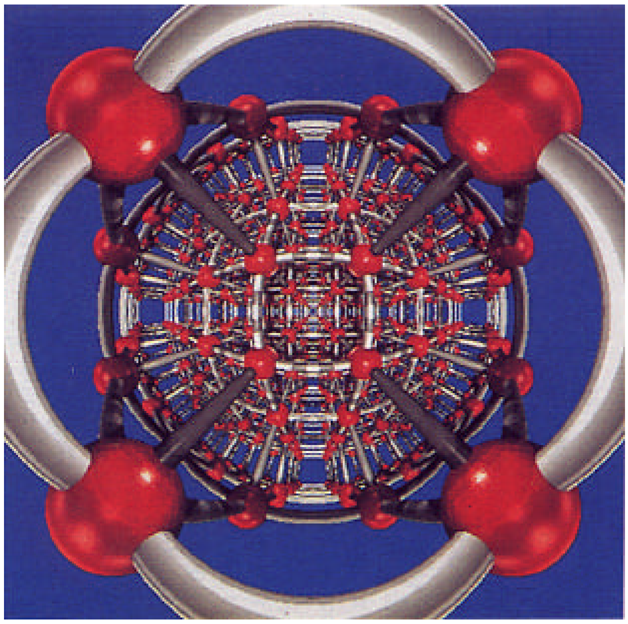
At rest



$v = 0.5\ c$



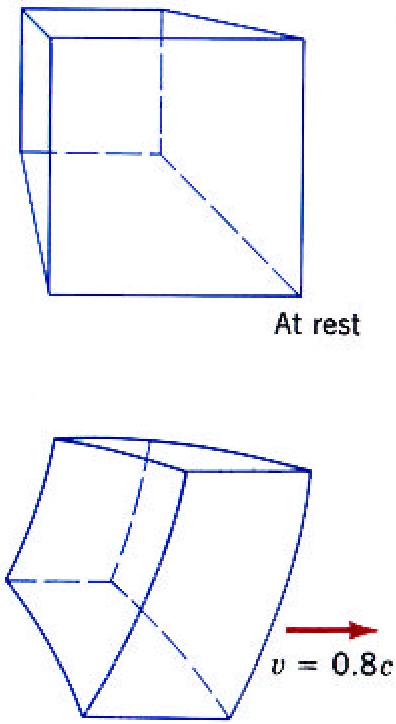
$v = 0.95\ c$



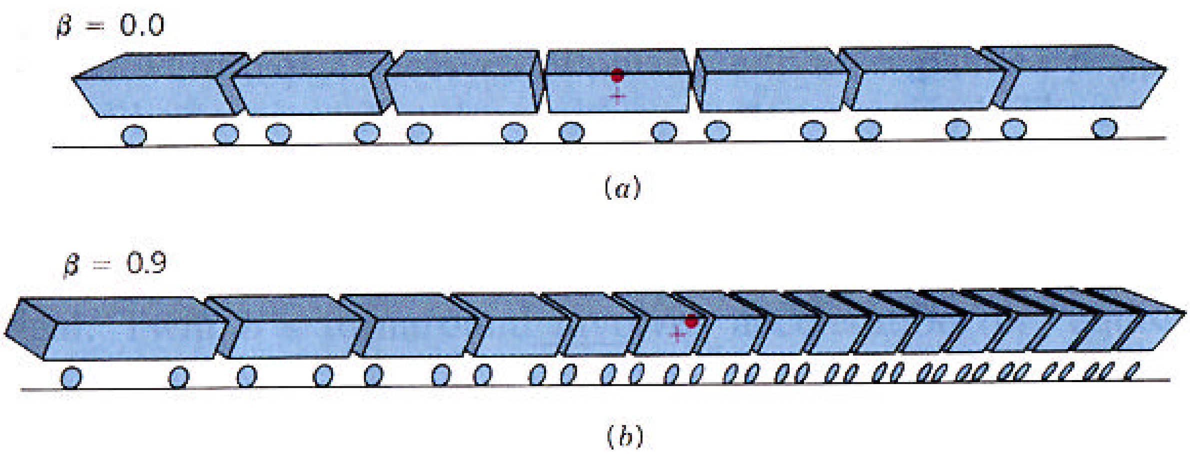
$v = 0.99\ c$

Computer generated graphics show the visual appearance of a three-dimensional lattice of rods and balls moving towards you at various speeds. The lattice only becomes distorted as v approaches c.

What Would You See?

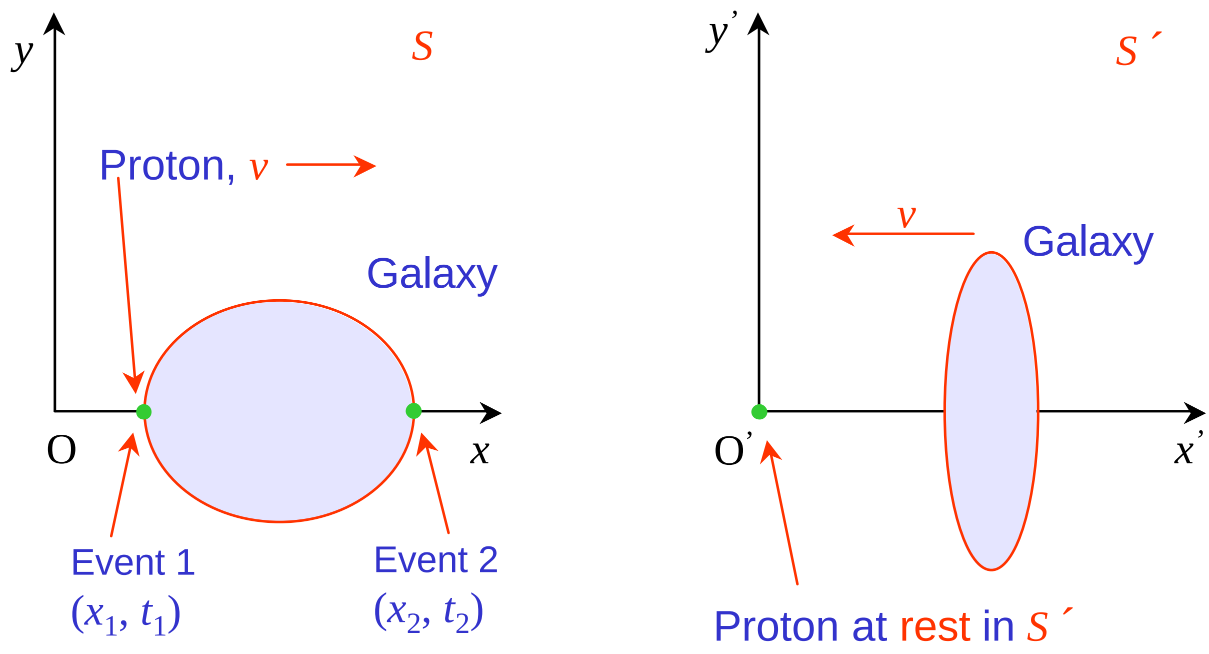


A view of a box at rest and a view of the box photographed by a single observer shows a distorted shape.



(a) The appearance of a train at rest as seen in a photograph. (b) The appearance of that train as it moves at 0.9*c* past a camera.

Galaxy Example



Diameter d (in S) = 10^5 light years

$E_{proton} = 10^{19}$ eV

M_p = 938 MeV/c^2

$\gamma = E/M_pc^2 = 1.066 \times 10^{10}$

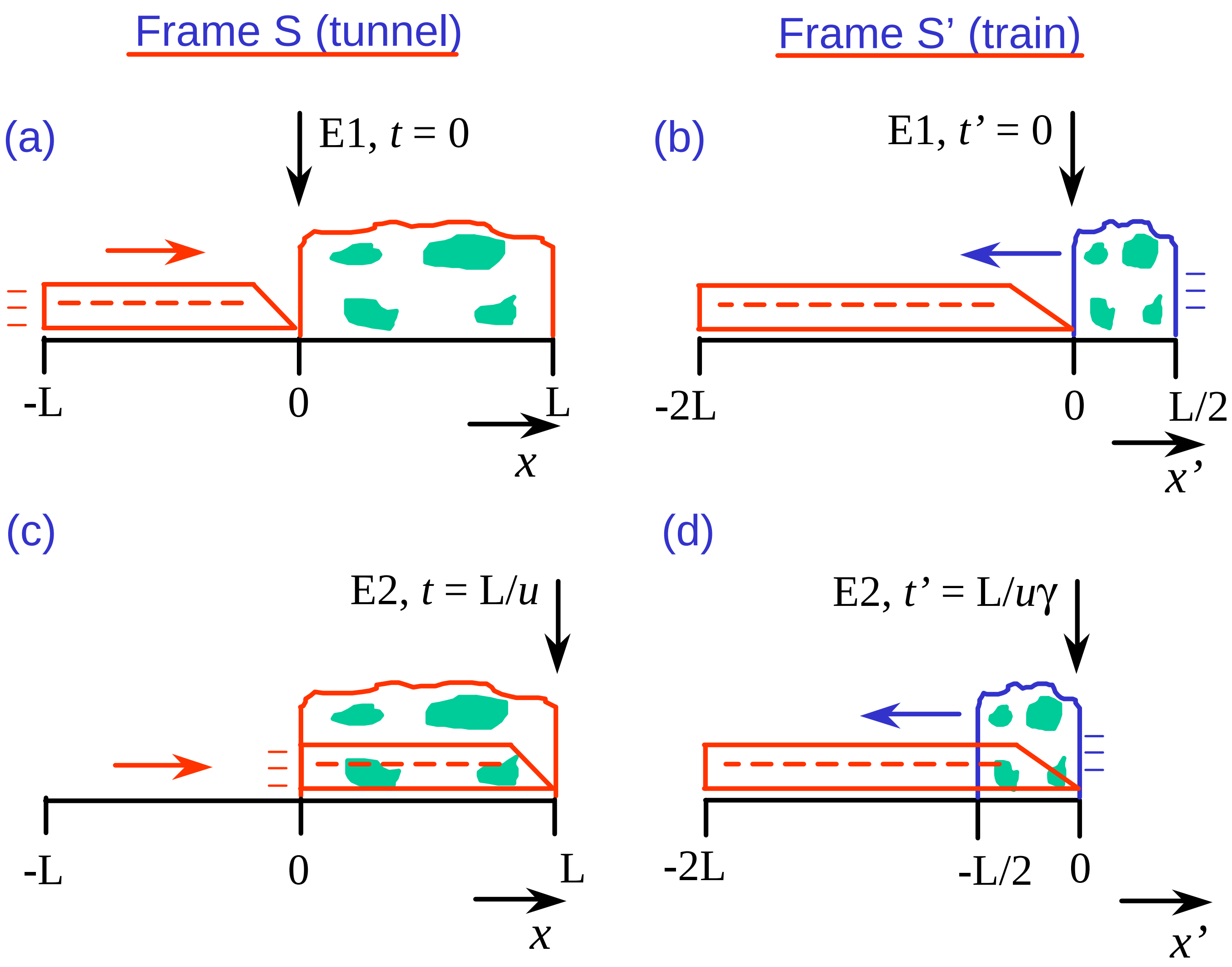
What is $\Delta t'$?

$\Delta t' = \Delta t/\gamma = 296$ s

$d' = d/g = 8.87 \times 10^{10}$ m

The Train in the Tunnel Problem

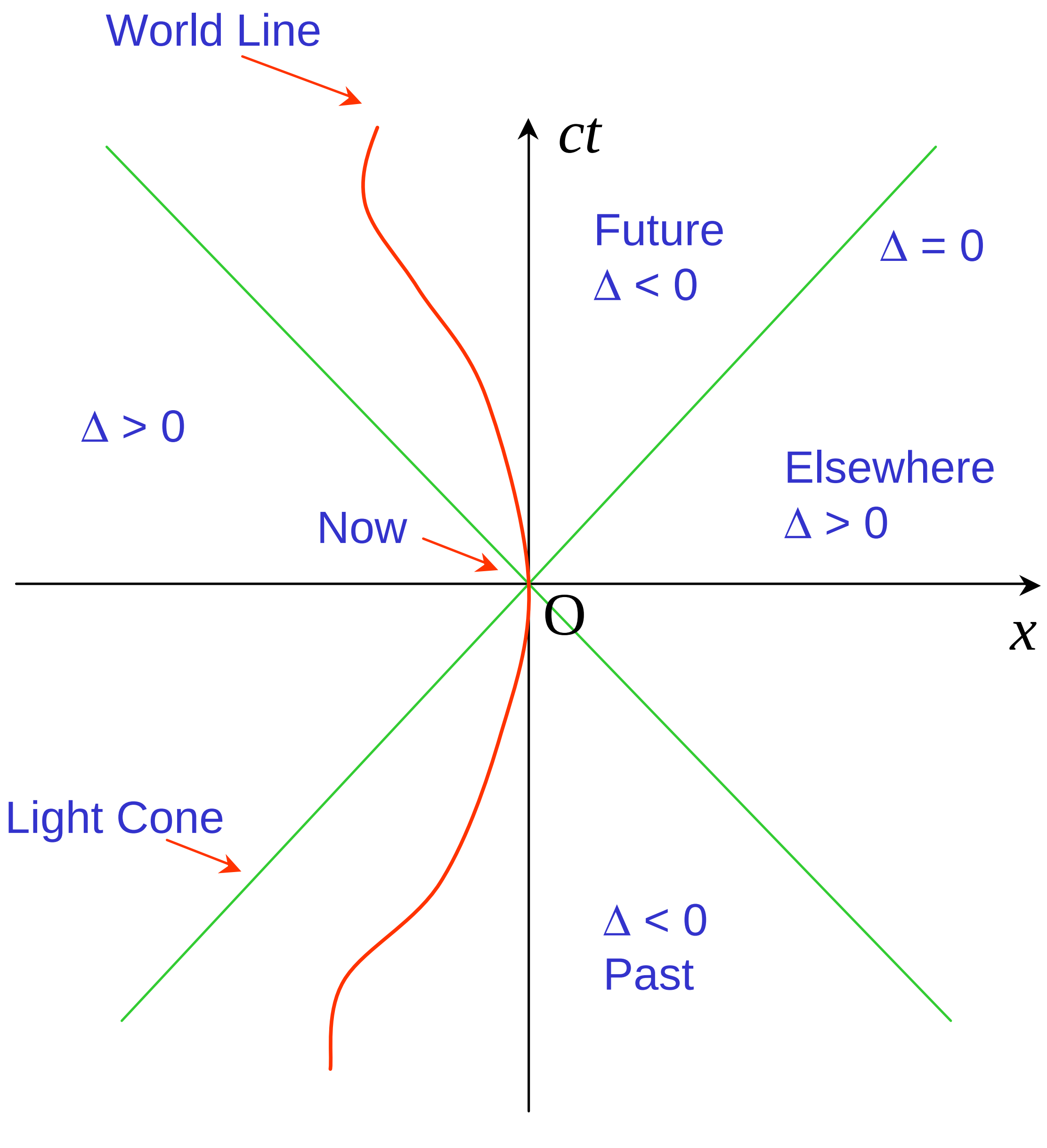
A train of proper length $2L$ = 500 m approaches a tunnel of proper length L = 250 m. The train's speed u is such that $\gamma = 2$. An observer at rest with respect to the tunnel measures the train's length to be contracted by a factor of 2 to 250 m and expects the whole train to fit in the tunnel. An observer on the train knows that the length of the train is 500 m, and that the tunnel is contracted by a factor of 2 to 125 m. Thus the observer on the train argues that the train will not fit into the tunnel. Who is right?



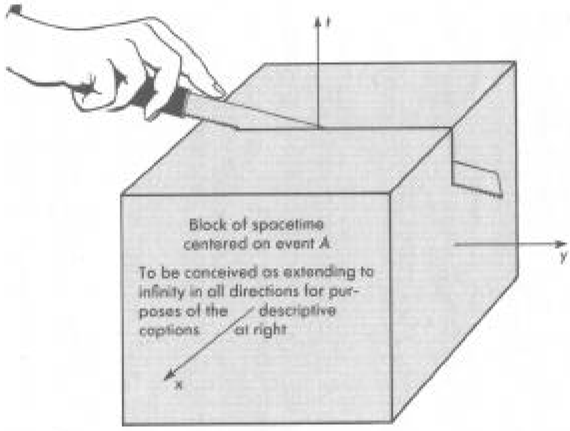
The train's speed u is such that $\gamma = 2$.

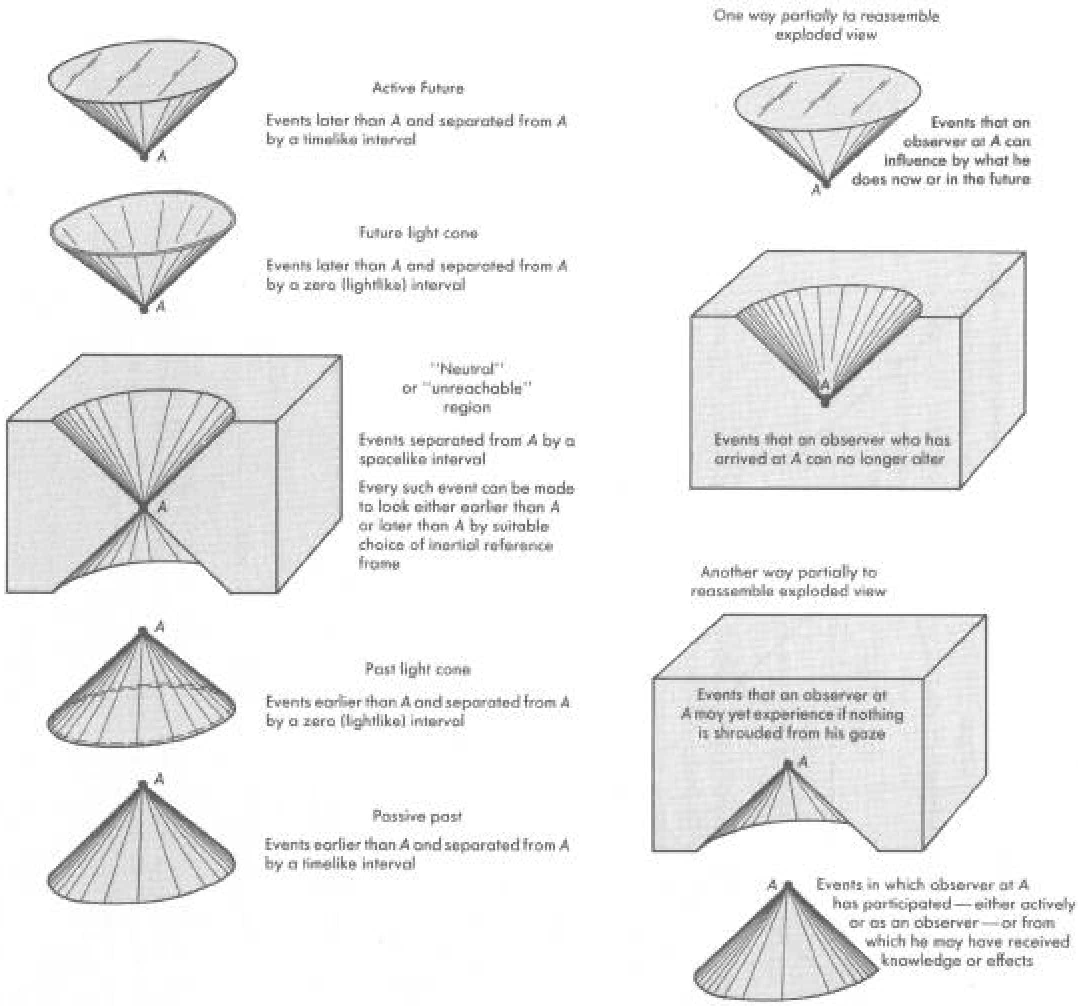
This problem demonstrates once more the importance of the concept of simultaneity in relativity theory.

The Light Cone

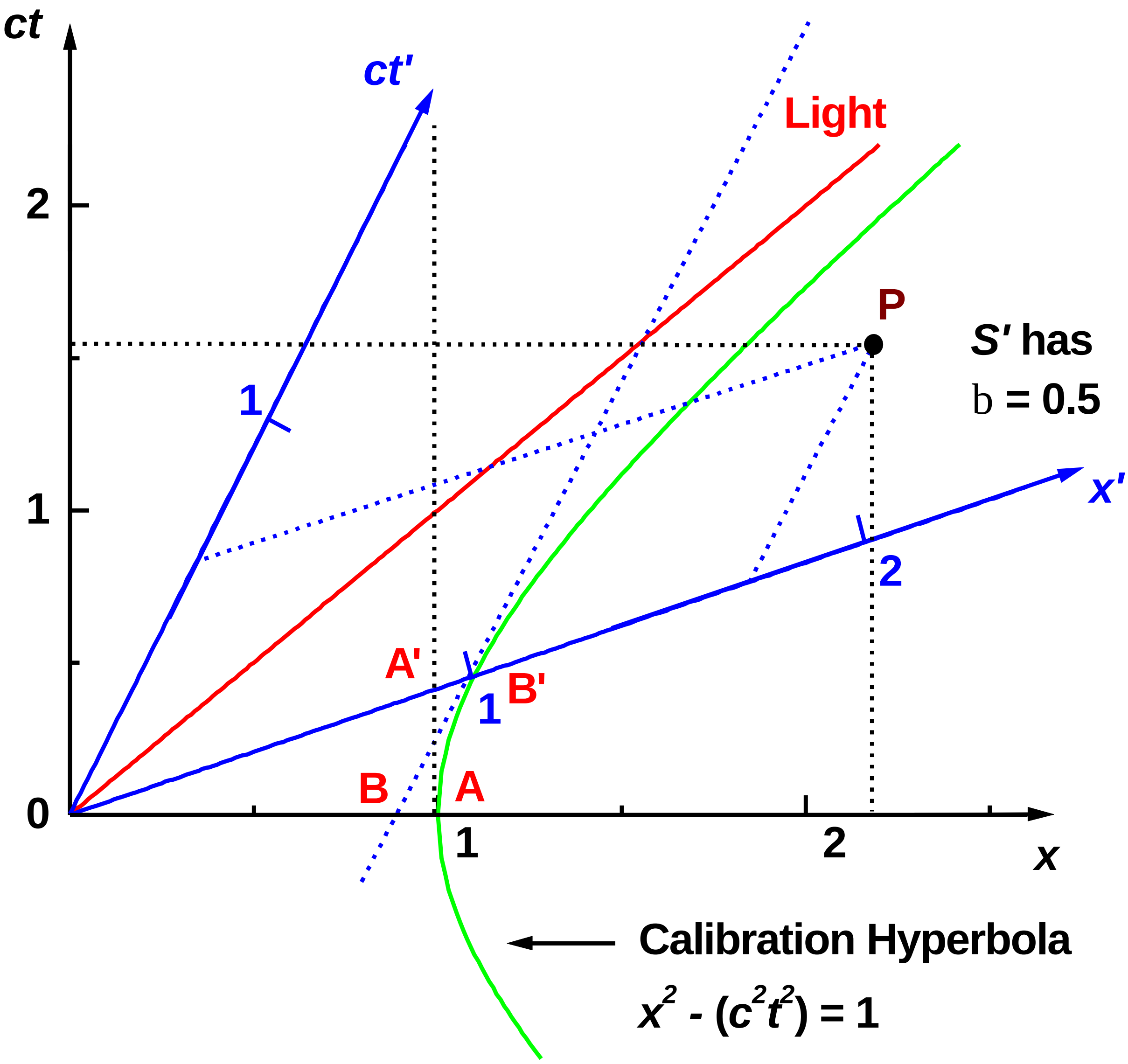


The Light Cone





Minkowski Diagram



Unit distance in *S* is OA

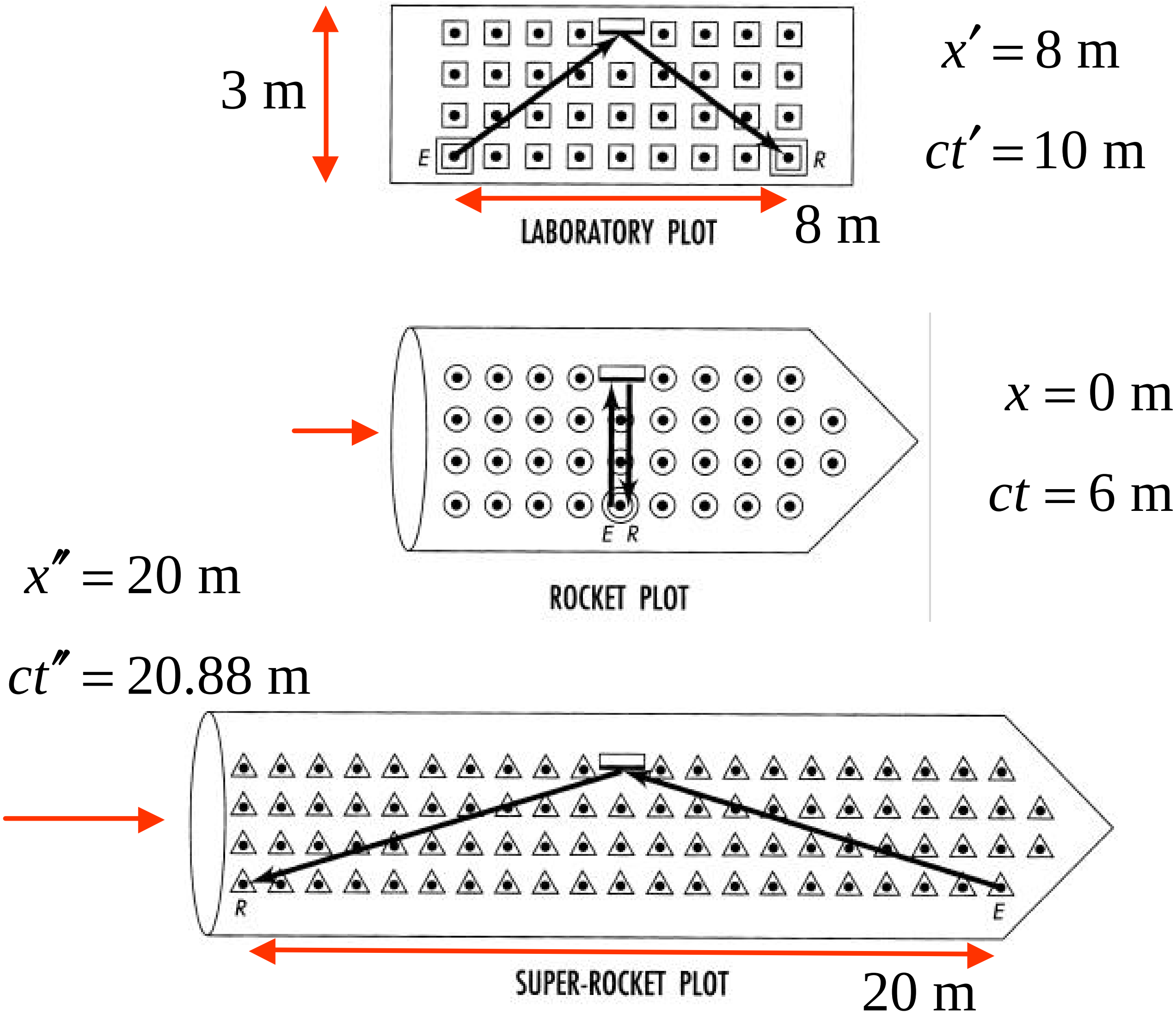
Unit distance in *S′* is OB'

Stick length OA' in *S′* < 1 (OB')

Stick length OB in *S* < 1 (OA)

Thus we have symmetry in the Lorentz contraction

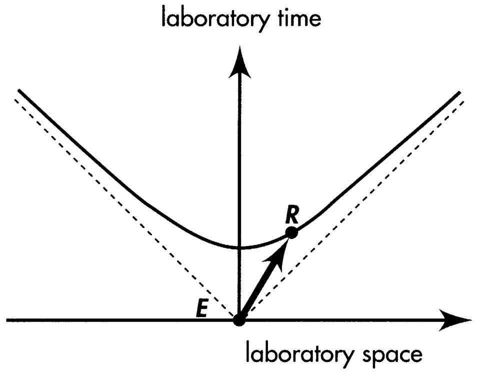
Relative to What?



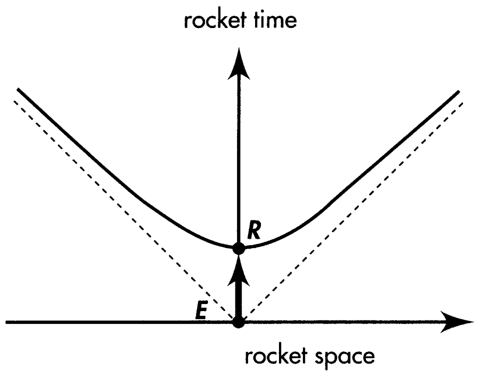
A flash path as recorded in three different inertial frames, showing emission and reception events after reflection at a mirror. The Super-Rocket moves to the right relative to the rocket.

Δ is INVARIANT

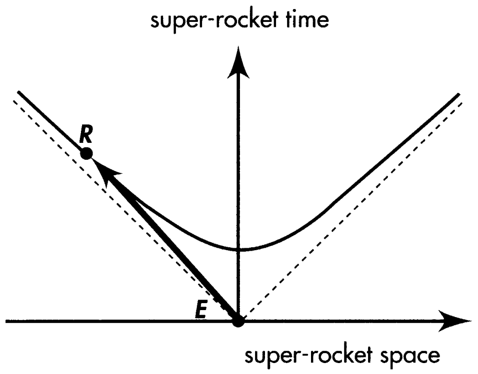
Spacetime Diagrams of the Same Event in Three Different Inertial Frames



LABORATORY
SPACETIME MAP



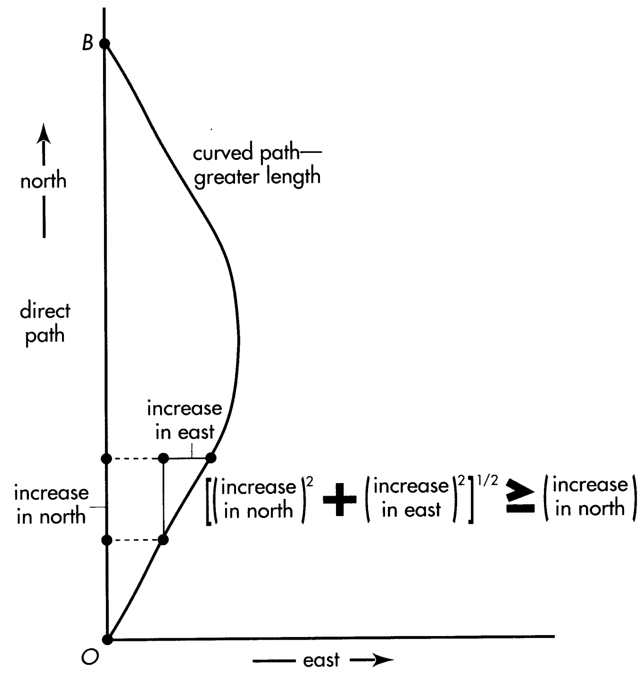
ROCKET
SPACETIME MAP



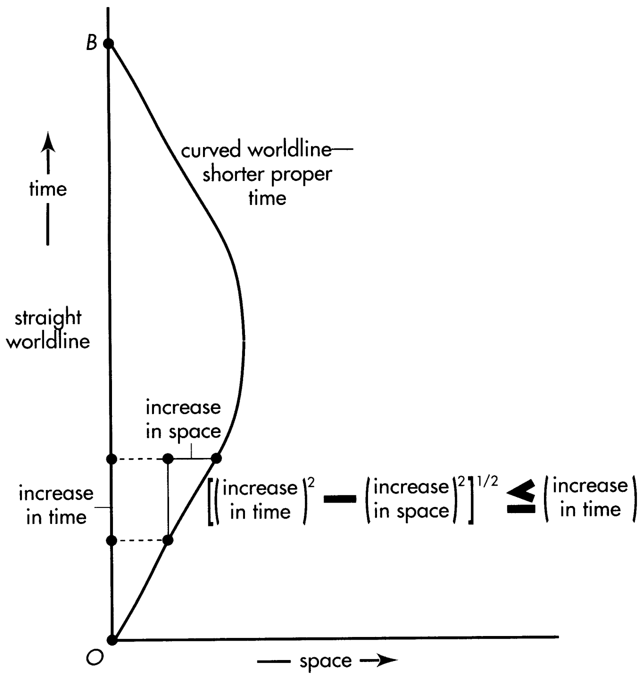
SUPER-ROCKET
SPACETIME MAP

The calibration hyperbola in each diagram satisfies the equation for the invariant interval (or proper time), which has the same value in all three inertial frames: $(\text{interval})^2 = (\text{space})^2 - (\text{time})^2$

Principle of Maximal Ageing



PATH IN SPACE



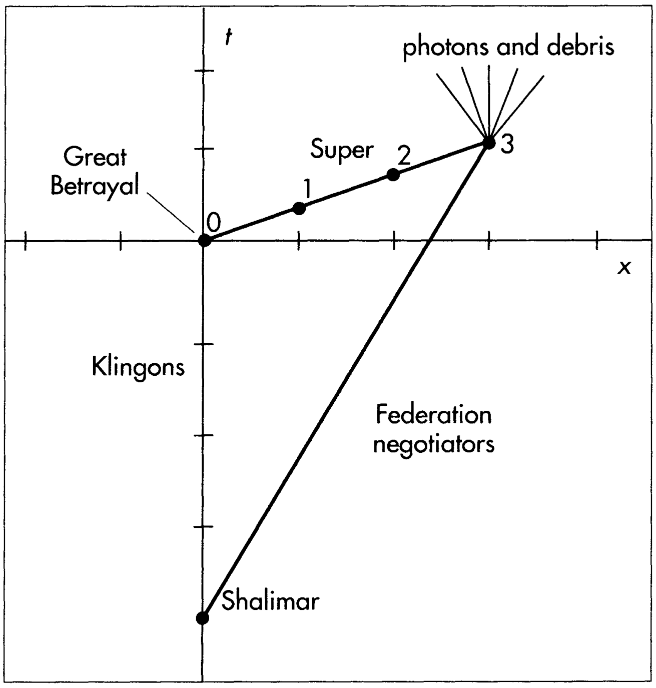
WORLDLINE IN SPACETIME

Path in space: In Euclidean geometry the curved path has greater length

Worldline in spacetime: In Lorentz geometry the curved worldline is traversed in the shorter proper time

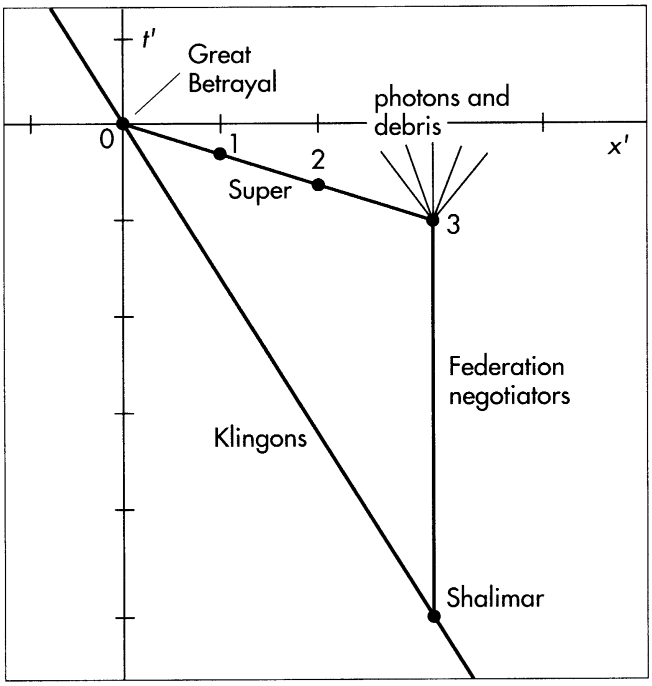
Causality – Faster Than Light?

Klingon ("laboratory") spacetime diagram



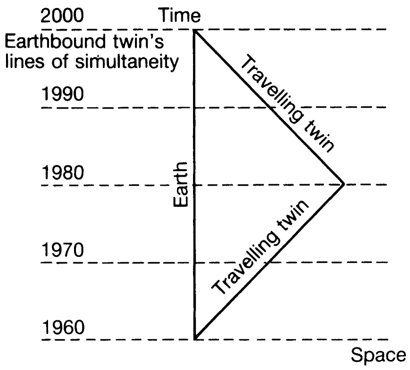
The Klingon worldline is the vertical time axis. The Treaty of Shalimar is followed four years later by the Great Betrayal (0) at which Klingons launch the Super, which moves at three times light speed. Travelling from left to right, the Super passes one Federation colony (1) and then another (2). Finally the Super destroys the retreating ship of Federation negotiators (3).

"Rocket" spacetime diagram of departing Federation negotiators

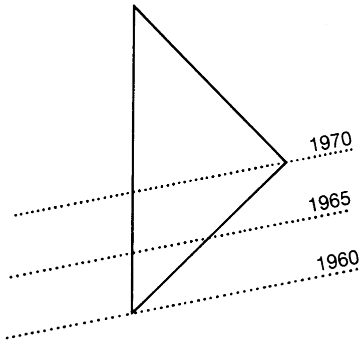


In this frame their destruction comes first (3), followed by the passage of the Super from right to left past Federation colonies in reverse order (2 followed by 1). Finally the Super enters the Klingon launcher without doing any further damage (0). Cause and Effect are violated!

The Twins Paradox

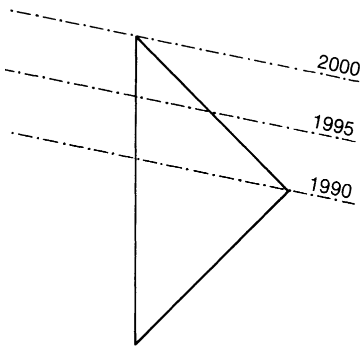


Earth-bound twin's lines of simultaneity

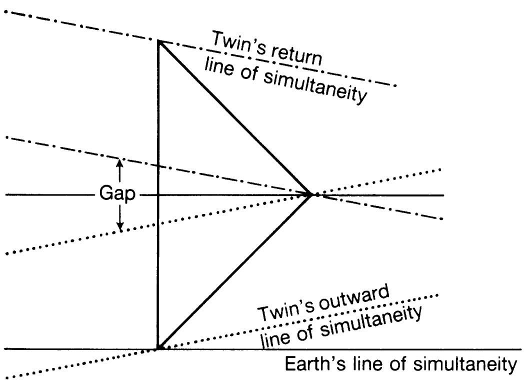


Travelling twin's lines of simultaneity on outward journey

$$\text{Here } g = 2$$

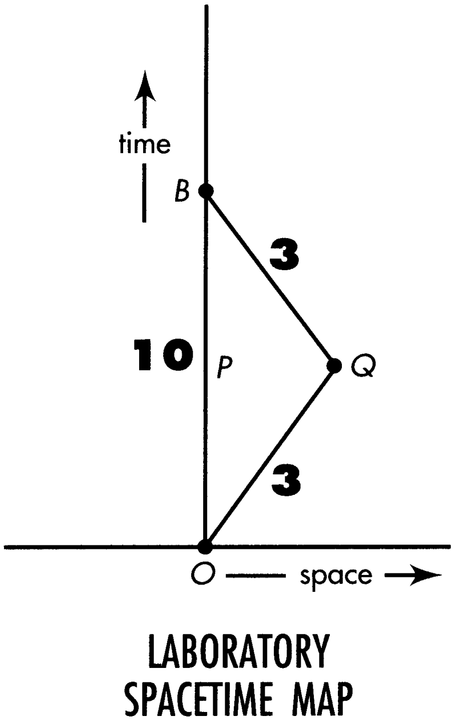


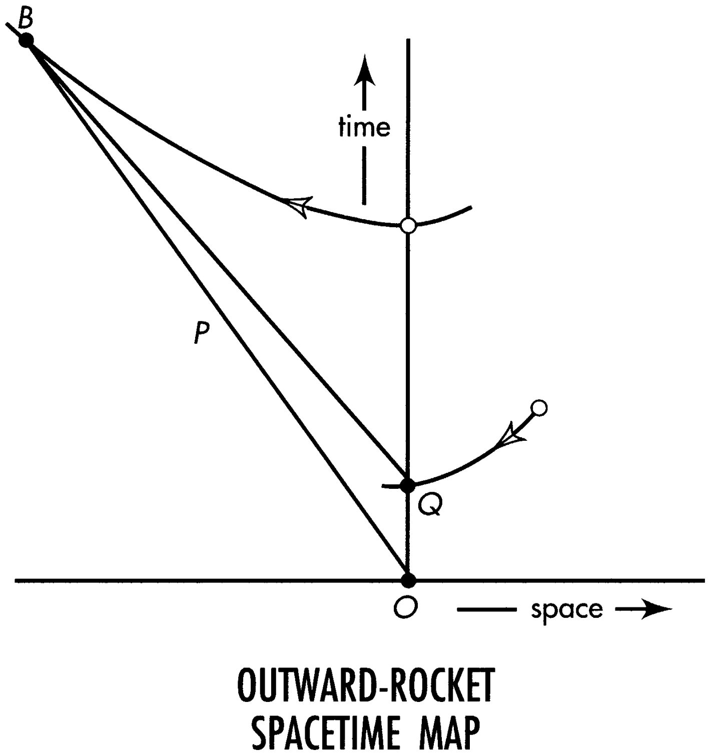
Travelling twin's lines of simultaneity on return journey



Travelling twin's lines of simultaneity on both outward and return journey

The Twins Again





Alternative worldlines (direct OPB and indirect OQB) between events O and B. Note that in the rocket rest frame we need two invariant hyperbolae to show how events Q and B transform. The direct worldline OPB has longer proper time – greater ageing – as computed using measurements from either frame.

Direct world lines show **maximal proper time**

The Speeding Rocket

Initial mass	=	m_i
Final mass	=	m_f
Exhaust speed	=	w
Final speed	=	u

We have:

$$\frac{m_i}{m_f} = \left[\frac{c+u}{c-u}\right]^{c/2w}$$

For w = 10 kms^{-1} we get, for $u = \frac{1}{2}c$:

$$\frac{m_i}{m_f} = 10^{7157}$$

But for $w \sim c$ we get:

$$\frac{m_i}{m_f} \approx \sqrt{3}$$