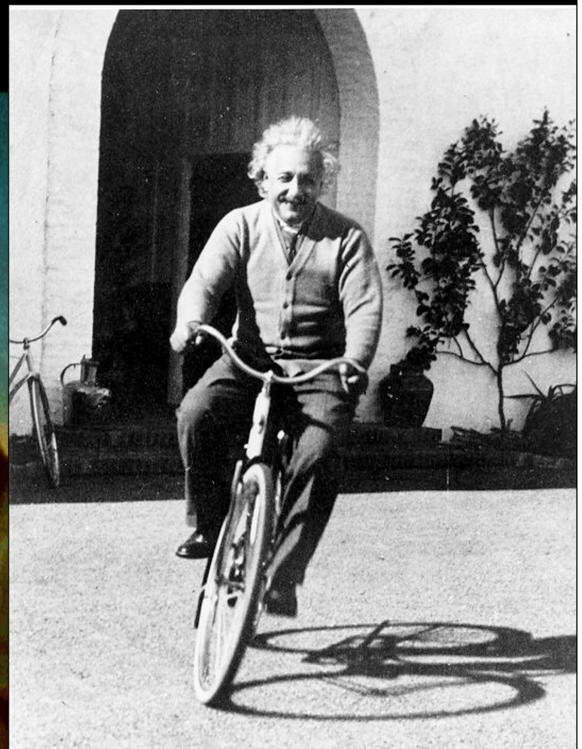
Special Relativity Lectures

First Year Lectures

Michaelmas Term

Dr. Robert A. Taylor



My Web page can be found at the following URL:

www.physics.ox.ac.uk/users/rtaylor

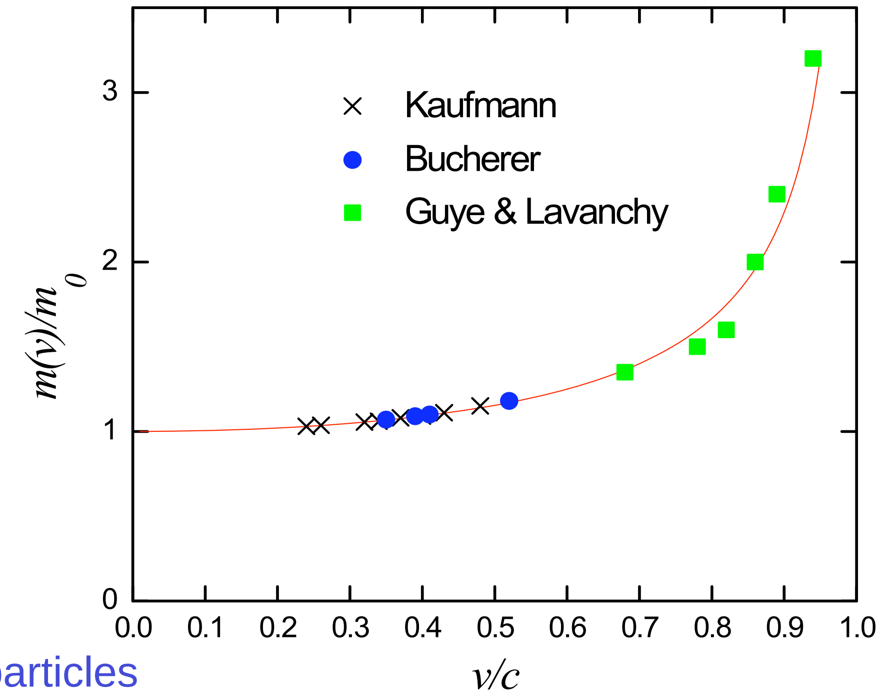
There are useful links there to other relativity sites.

It is only available within the Oxford domain.

I have a Horizon video about

Einstein and Relativity – would you like to see it?

Velocity Dependence of Inertial Mass

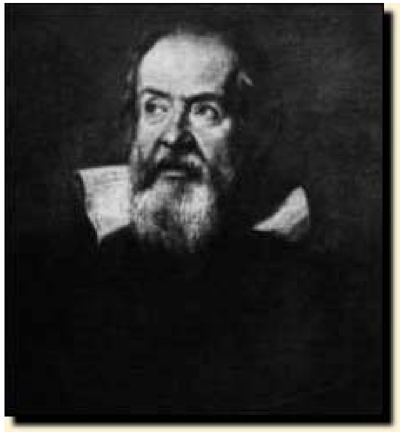


Variation of inertial mass with speed for electrons. Based on the data of Kaufmann (1910), Bucherer (1909) and Guye and Lavanchy (1915).

- Easy to accelerate
- Electrons are the lightest material particles

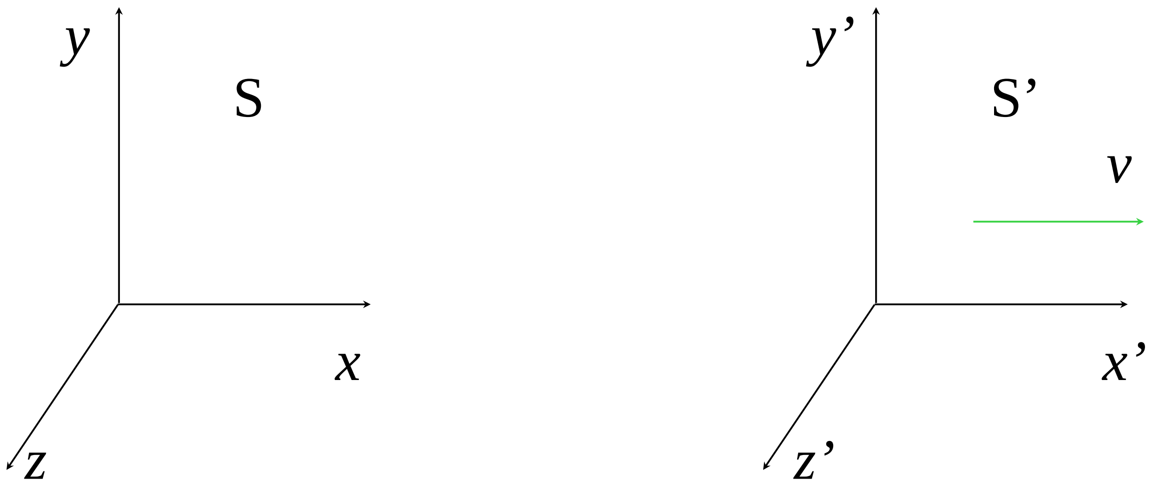
$$m_e = 0.5\mathrm{MeV}/c^2 \qquad m_e = \frac{1}{1840} m_p$$

- Can measure ***p*** by bending in a magnetic field
- Or *E* from acceleration voltage
- Measure ***v*** by time-of-flight



Galileo Galilei *(Italy, 1564-1642) rethought traditional ideas about force and motion, ideas that were plausible (and had been upheld by all authorities for many centuries) but were in fact misleading. Using his own experiments he created a new science of mechanics.*

Galilean Relativity



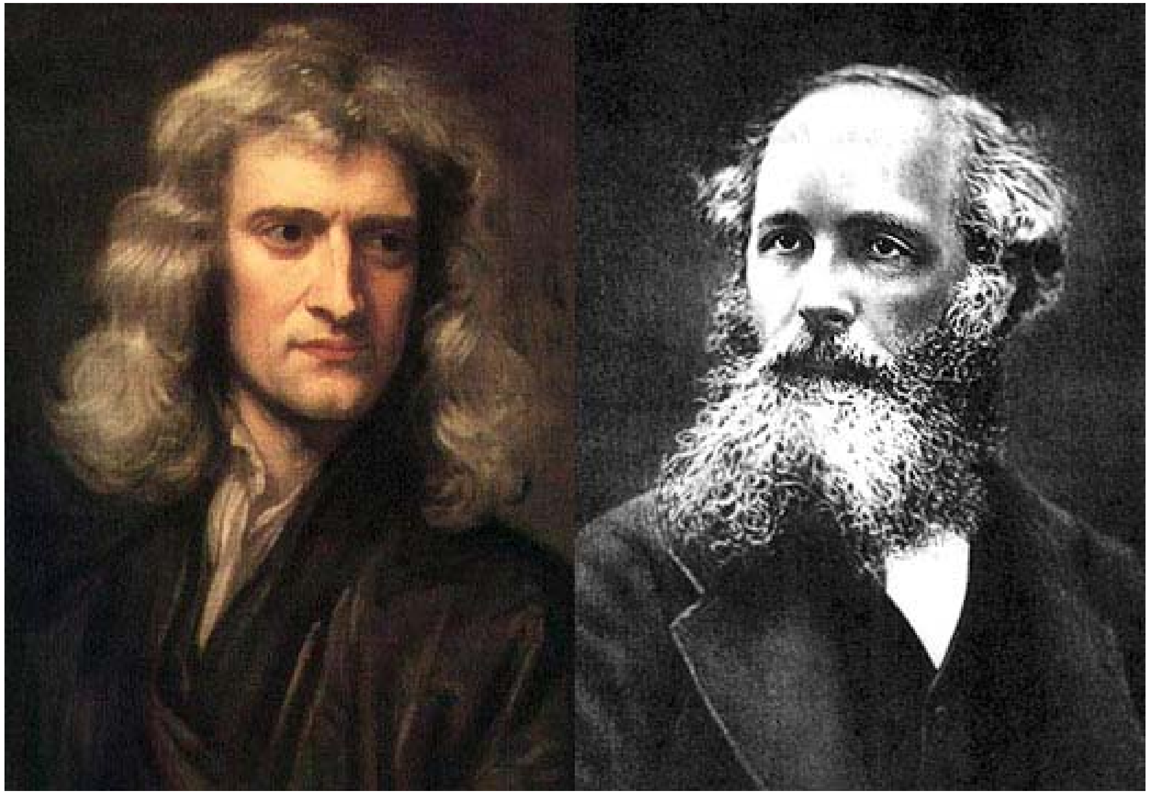
$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{S'} = v \qquad \left.\frac{\mathrm{d}x'}{\mathrm{d}t'}\right|_{S} = -v$$

$$x' = x - vt \qquad t' = t$$

$$x = x' + vt' \qquad t = t'$$

These are the Galilean transformations

In general: $\mathbf{u}' = \mathbf{u} - \mathbf{v}$



Einstein realized that the world described by Isaac Newton (left), in which one could add and subtract velocities, and that described by James Clerk Maxwell, in which the speed of light is constant, could not both be right. He became obsessed by the problem—and special relativity was the result.

The Problem with Light

Maxwell's equations in free space are $\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$ and $\nabla^2 \mathbf{H} = \frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2}$

These are wave equations with solutions $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k}.\mathbf{x} - \omega t)$ $\quad \omega = ck, \; k = 2\pi/\lambda$

The phase is defined as $\varphi = (\mathbf{k}.\mathbf{x} - \omega t)$ and is an invariant.

For a Galilean transformation with $\mathbf{x}$ along $\mathbf{k}$ we have

$$\left(\frac{\partial x}{\partial t}\right)_\varphi = \frac{\omega}{k} = c \qquad \mathbf{k}'.\mathbf{x}' - \omega' t' = \mathbf{k}.\mathbf{x} - \omega t$$

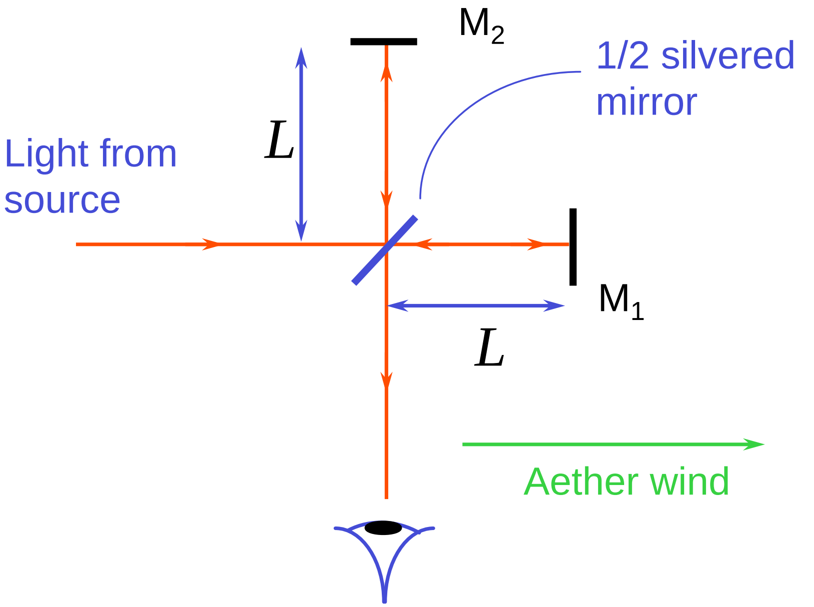
$$\mathbf{k}'.(\mathbf{x} - \mathbf{v}t) - \omega' t' = \mathbf{k}.\mathbf{x} - \omega t$$

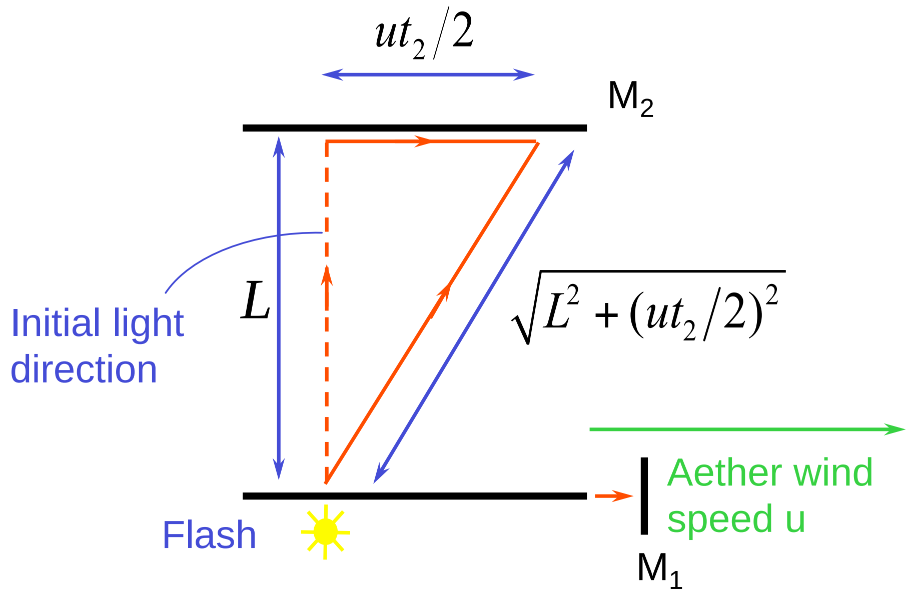
$$\mathbf{k}' = \mathbf{k}, \quad \omega' = \omega - \mathbf{v}.\mathbf{k}' = \omega - \mathbf{v}.\mathbf{k} = \omega\left(1 - \frac{\mathbf{v}.\hat{\mathbf{k}}}{c}\right)$$

And the phase velocity in S' is $c' = \frac{\omega'}{k'} = c - \mathbf{v}.\hat{\mathbf{k}} \neq c$

Michelson - Morley Experiment

ALBERT
ABRAHAM
MICHELSON
1852-1931



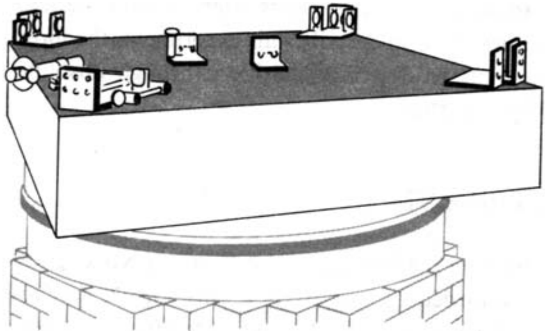


$$\boxed{c\Delta t = \frac{Lu^2}{c^2}}$$

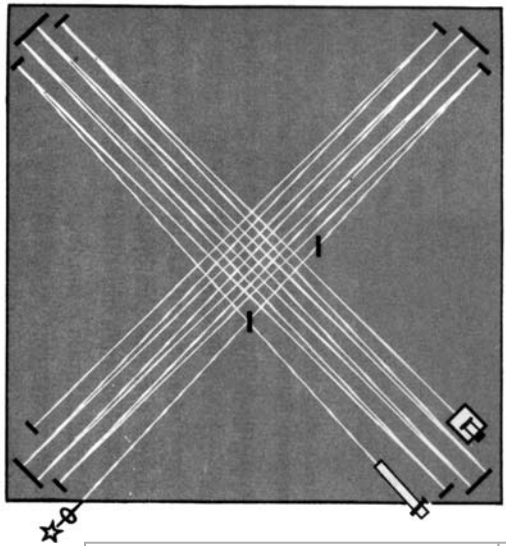
$$t_1 = \frac{L}{c+v} + \frac{L}{c-v}$$

Michelson - Morley Experiment

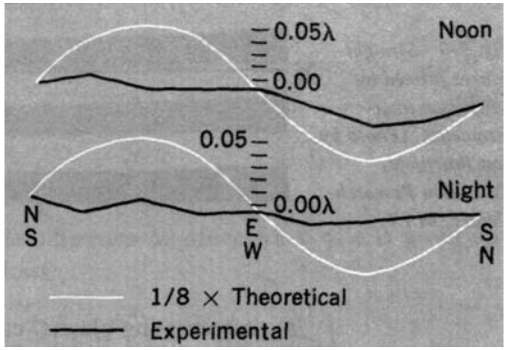
(a)



(b)



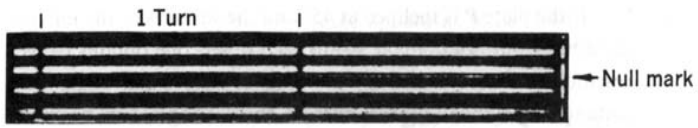
(c)



(a) A sketch of the apparatus

(b) Plan view of the optics

(c) Variation of fringe position during one rotation



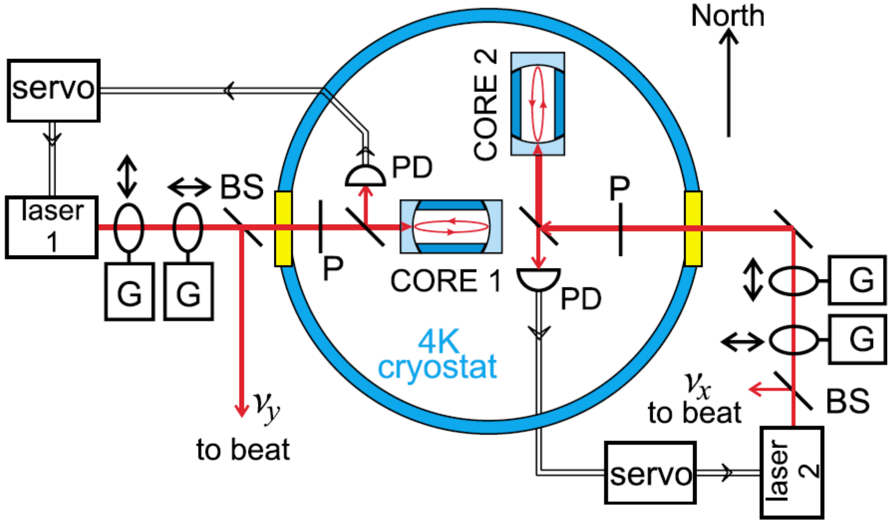
Observer, year	L /cm	δ_{calc}	δ_{obs} (upper lim.)	Ratio
Michelson and Morley, 1887	1100	0.40	0.01	40
Morley and Miller, 1902-04	3220	1.13	0.015	80
Miller, 1923-24	3220	1.12	0.03	40
Kennedy, 1926	200	0.07	0.002	35
Illingworth, 1927	200	0.07	0.0004	175
Michelson et al, 1929	2590	0.90	0.01	90
Joos, 1930	2100	0.75	0.002	375

Modern Michelson - Morley Experiment using Cryogenic Optical Resonators

Holger Müller et al, Phys. Rev. Lett. 91, 020401, (2003)

Anisotropy in speed of light in the two resonators is

$$\frac{\Delta c}{c} = (2.6 \pm 1.7) \times 10^{-15}$$



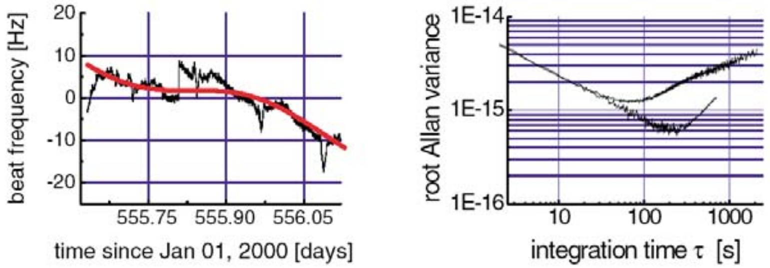
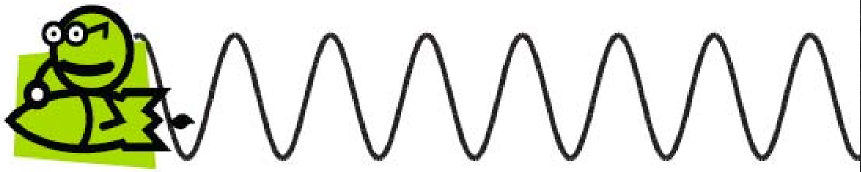


FIG. 2 (color online). Left: Typical data set fitted with a 12 h sinewave amplitude, a linear drift, and a constant offset. Peaks occur every few hours due to automatic LN2 refills. Right: Root Allan variance calculated from this data (upper curve), and from a quiet part between two LN2 refills (118 min starting at 555.87 days; lower curve).

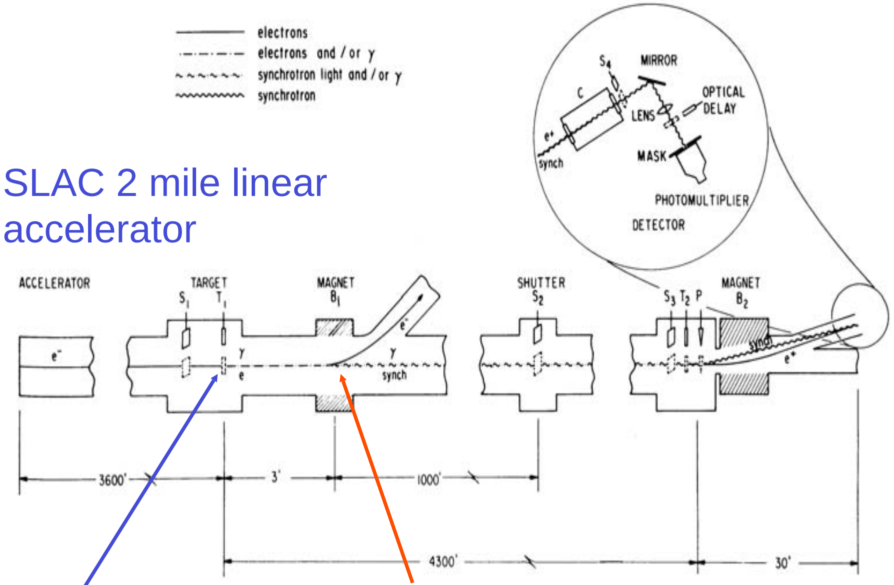


A streetcar trundles below the clock tower in Bern that Einstein made famous with his thought experiment about racing a light beam.

Velocity of Light vs Frequency

(Brown et al. PRL **30**, 763, 1973)

SLAC 2 mile linear accelerator



Target to produce 7GeV γ rays by Bremsstrahlung

Velocity of γ's measured by time-of-flight technique over 4300' – S.I.?

Result:

$$\frac{c(7\ \mathrm{GeV}) - c(\mathrm{eV})}{c(\mathrm{eV})} = (1.8 \pm 6)\times 10^{-6}$$

Also for electrons at 11 GeV vs c we have:

$$\frac{(c - v_{\mathrm{e}})}{c} = (-1.3 \pm 2.7)\times 10^{-6}$$

From Special Relativity, we would expect:

$$\frac{(c - v_{\mathrm{e}})}{c} \approx \frac{M_{\mathrm{e}}^2 c^4}{2E_{\mathrm{e}}^2} \approx 10^{-9}$$

Einstein at the age of 5 in 1884

Einstein Young and Old

I Einstein at his desk in the Patent office,
Bern, ca. 1905.
(Einstein Archive, Courtesy AIP Niels Bohr Library)

XII The last known picture of Einstein,
taken in March 1955, in front
of 112 Mercer Street.

"Instinct says beer, Reason says Carlsberg."
(Italian advertisement, 1970s.)

Einstein - The Man

- Born: March 14^{th}, 1879 in Ulm
- In 1900 qualified as a teacher in maths and physics
- Became a Swiss citizen in 1901
- 1902 moved to Federal Patent Office in Bern
- 1903-4 Papers on statistical mechanics
- 1905 Nobel Prize paper on light quanta (awarded 1921)
- 1905 Ph.D. from Zurich on statistical mechanics
- 1905 Special relativity paper
- 1905 - quite a year!

History of Relativity Theory

- 1895 Lorentz derives force on a charged particle
- November 1887 - Michelson-Morley experiment
- Kelvin, Rayleigh and Lorentz were disappointed
- Poincaré - questioned simultaneity in 1898!
- Voigt, Lorentz write down the Lorentz Trans. (1904)
- Fitzgerald puts forward contraction hypothesis
- Poincaré derives transformation of velocity (1905)
- Einstein states the two principles of relativity (1905)

Einstein's two postulates of the Special Theory of Relativity

- The laws of physics are the same in all inertial frames
- The speed of light in empty space is the same in all inertial frames and is independent of the motion of its source

What is an Inertial Frame?

- Set up a lattice of identical clocks, each separated by the same, KNOWN distance L
- Send a flash of light from the reference clock at the origin. This travels at the speed of light c
- When each clock receives the flash it immediately knows its position in space-time
- We now have an INERTIAL FRAME