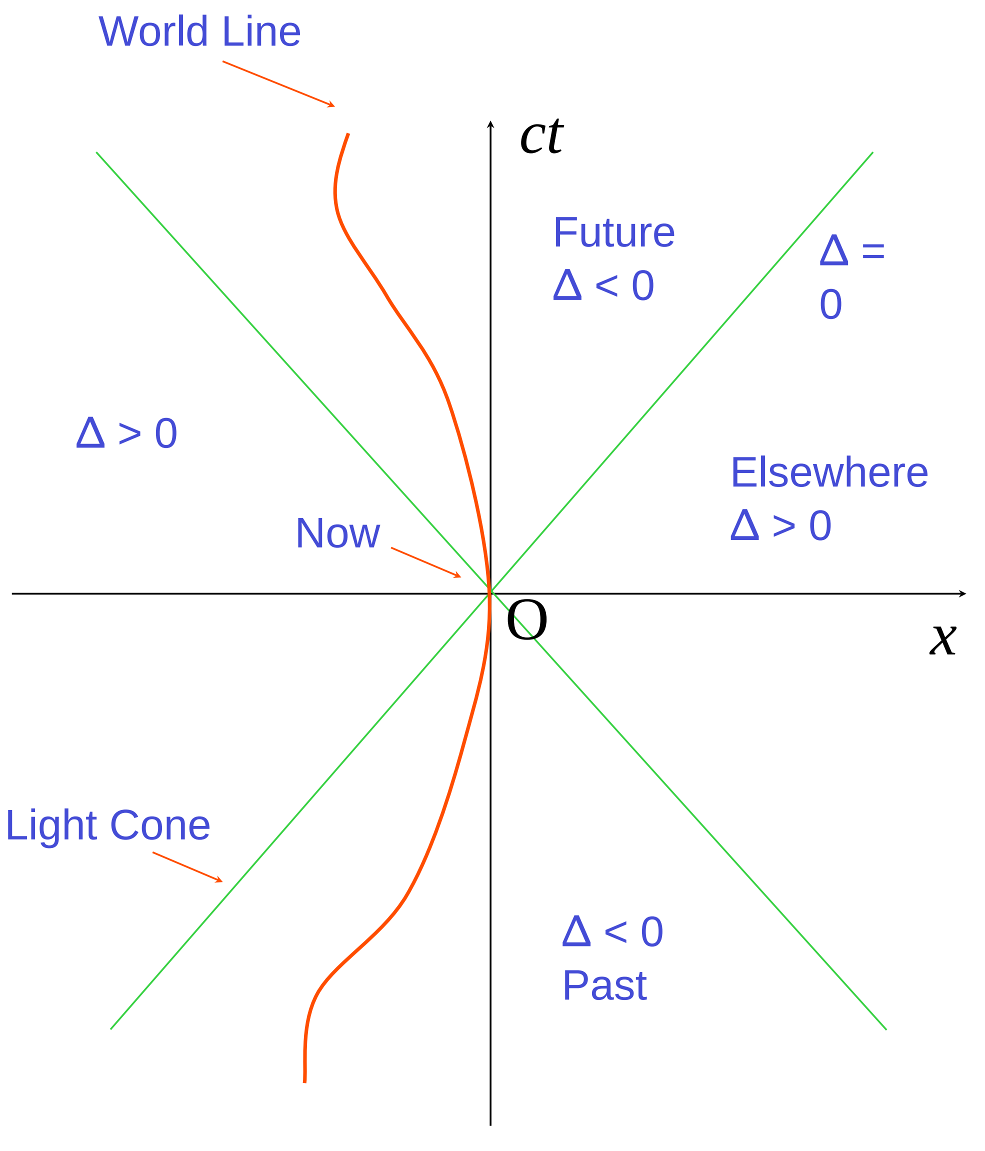
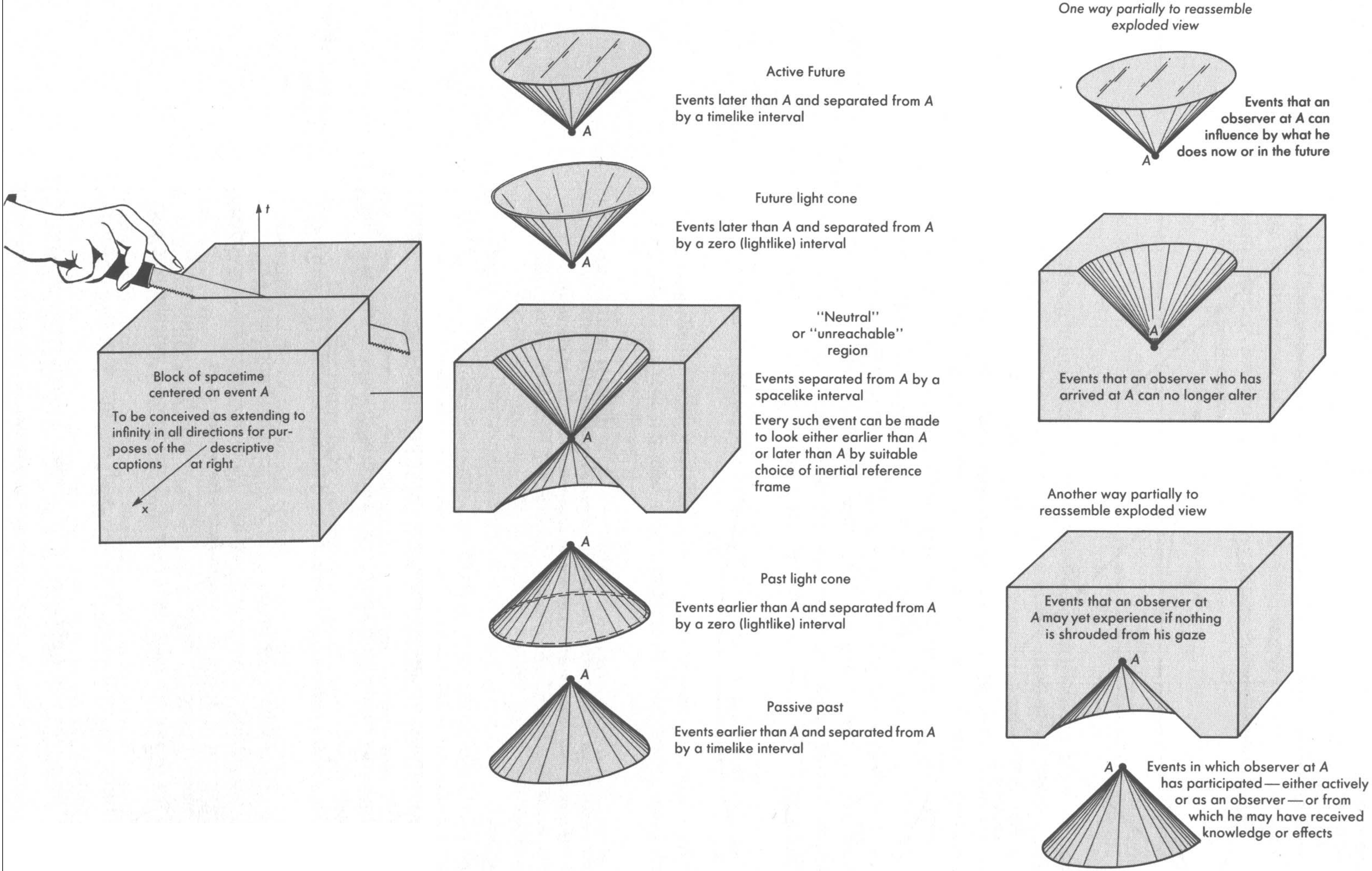
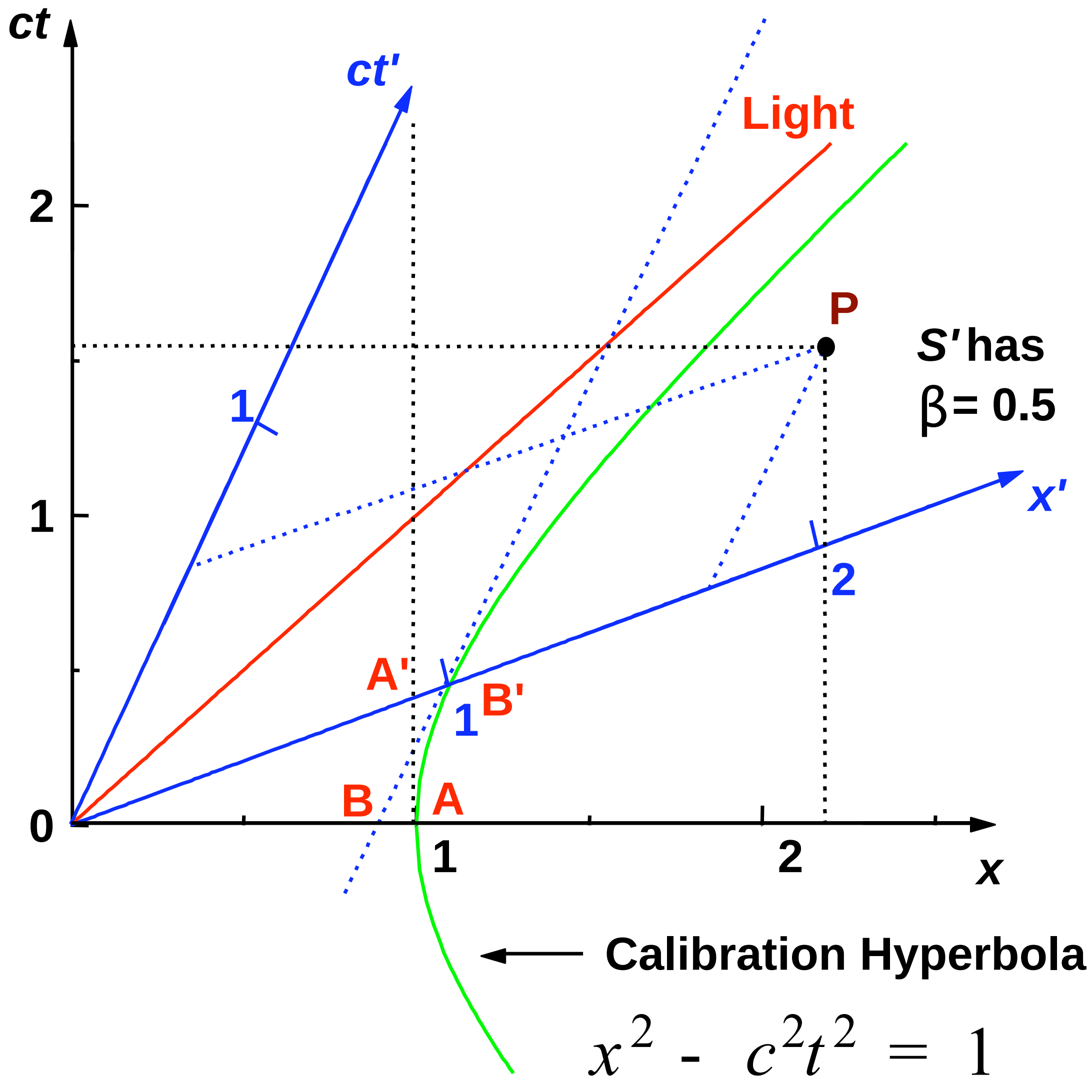
The Light Cone



The Light Cone



Minkowski Diagram



Unit distance in *S* is OA

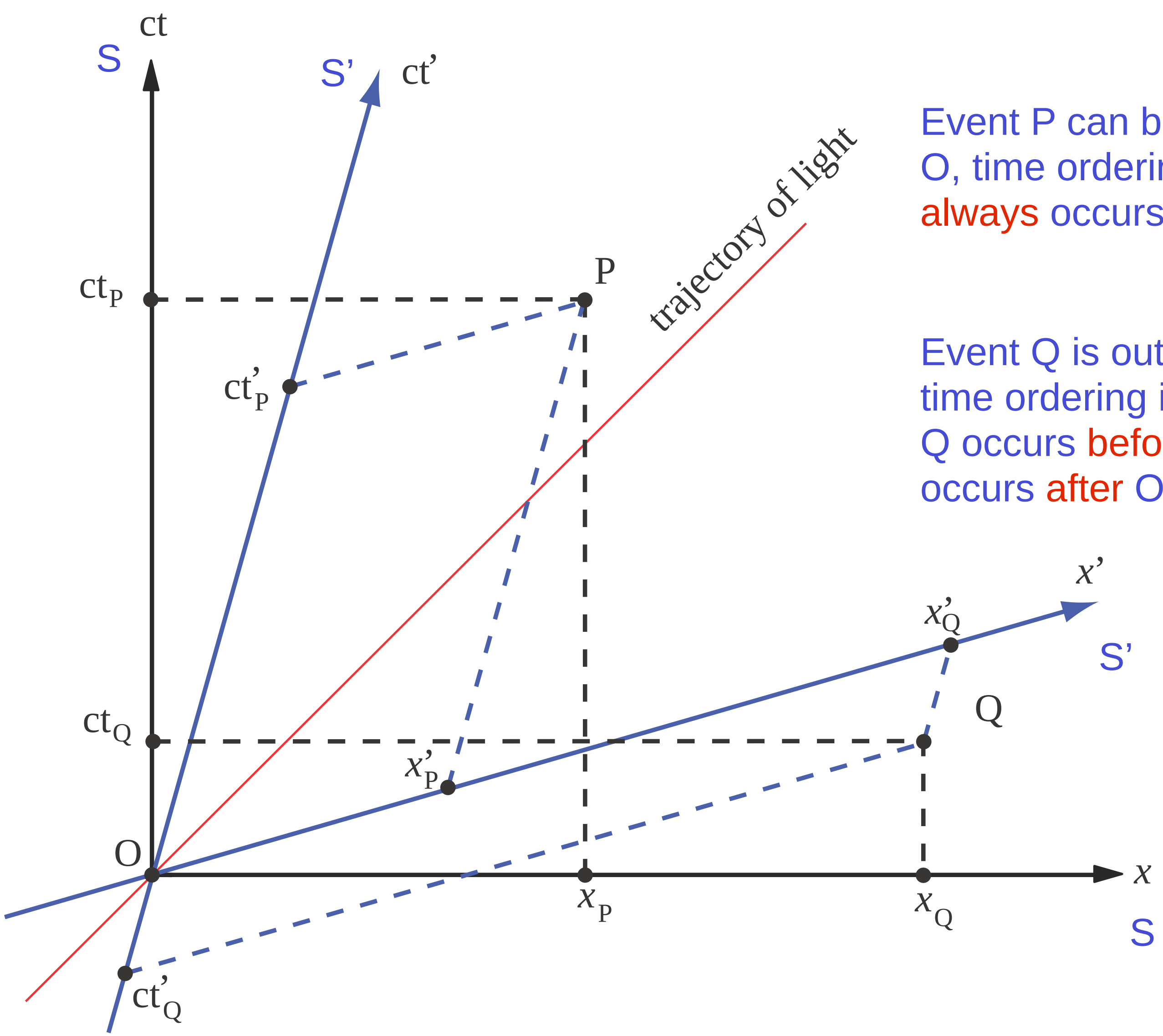
Unit distance in *S*' is OB'

Stick length OA' in *S*' < 1 (OB')

Stick length OB in *S* < 1 (OA)

Thus we have SYMMETRY in the Lorentz contraction

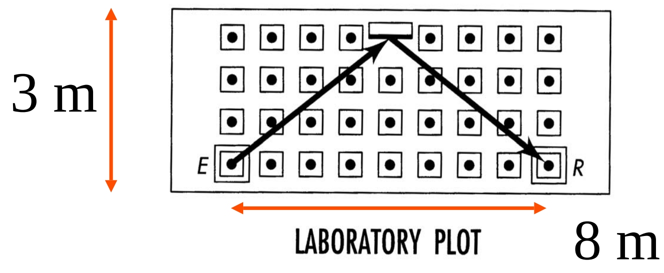
Minkowski Diagram explaining light cone



Event P can be influenced by event O, time ordering is preserved. P always occurs after O in all frames.

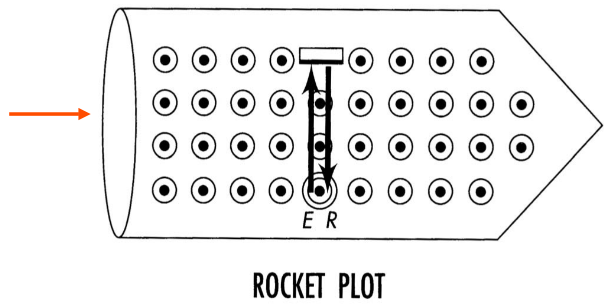
Event Q is outside the light cone and time ordering is not preserved. In S', Q occurs before O, whilst in S, Q occurs after O.

Relative to What?



$x' = 8$ m

$ct' = 10$ m

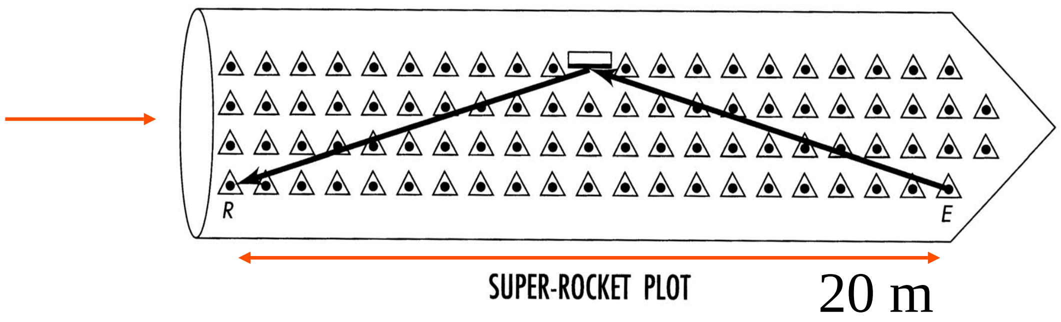


$x = 0$ m

$ct = 6$ m

$x'' = 20$ m

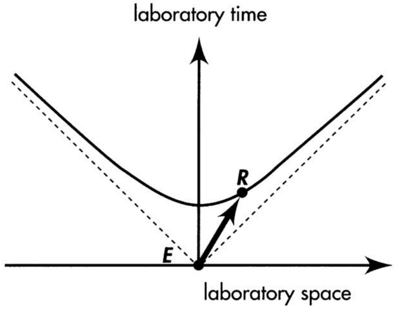
$ct'' = 20.88$ m



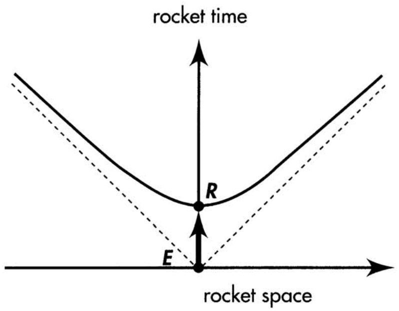
A flash path as recorded in three different inertial frames, showing emission and reception events after reflection at a mirror. The Super-Rocket moves to the right relative to the rocket.

Δ is **INVARIANT**

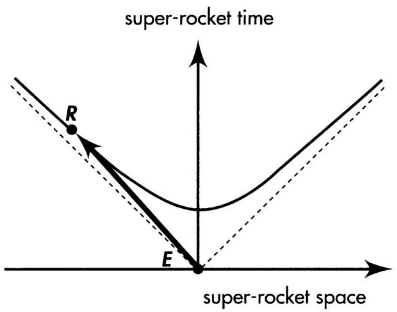
Spacetime Diagrams of the Same Event in Three Different Inertial Frames



LABORATORY SPACETIME MAP



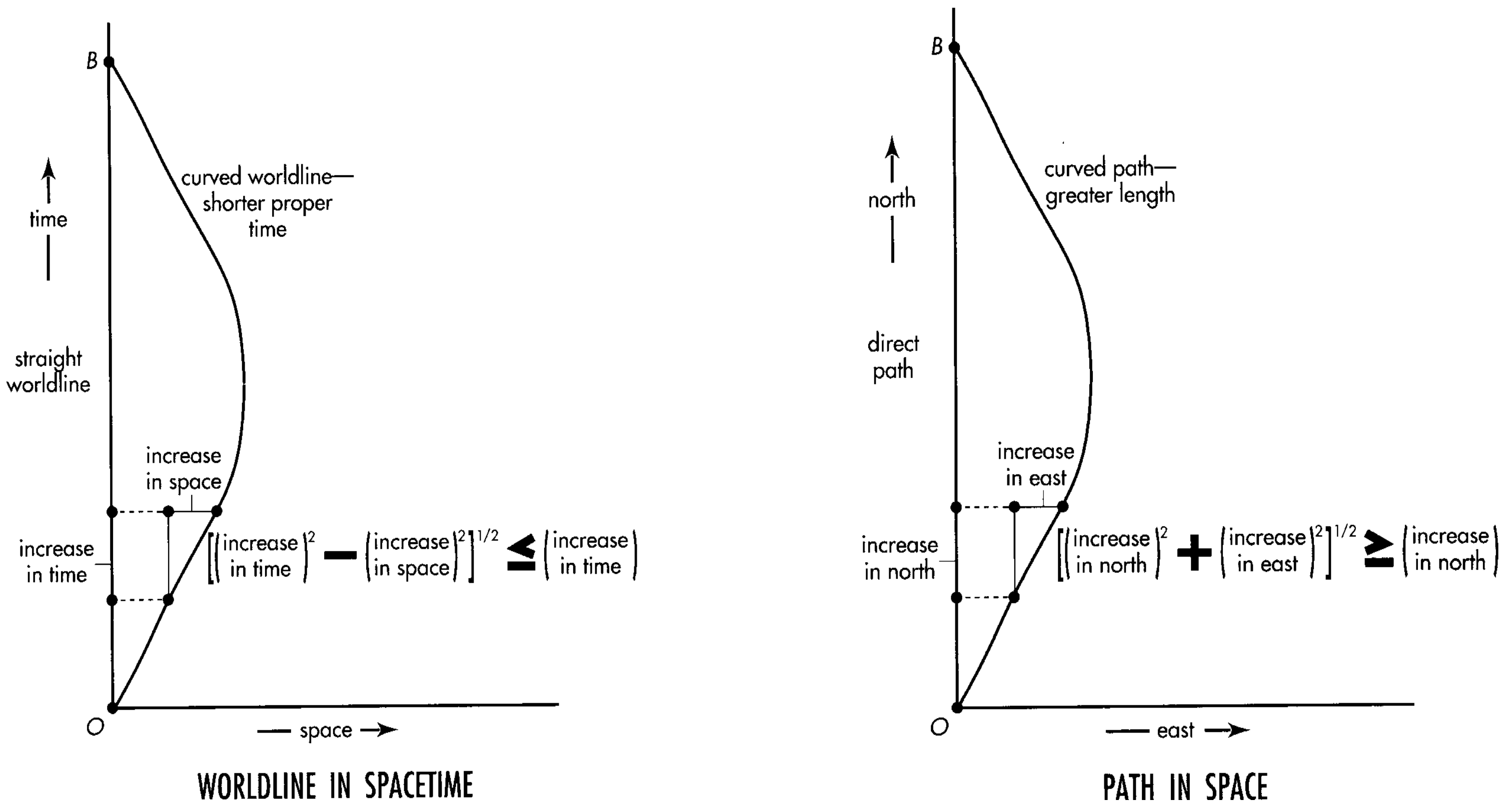
ROCKET SPACETIME MAP



SUPER-ROCKET SPACETIME MAP

The calibration hyperbola in each diagram satisfies the equation for the invariant interval (or proper time), which has the same value in all three inertial frames: $(\text{interval})^2 = (\text{space})^2 - (\text{time})^2$

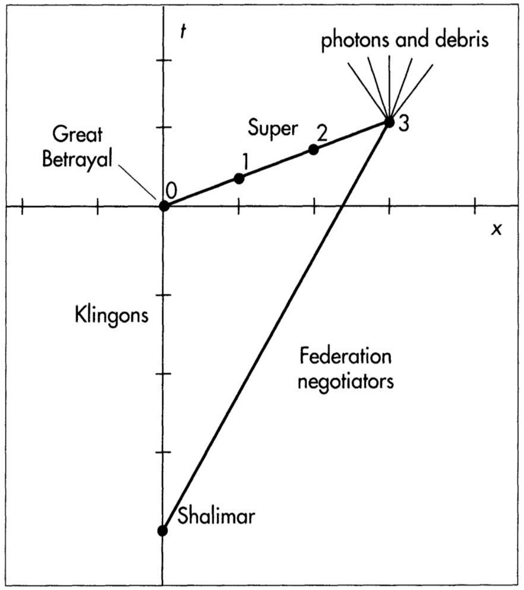
Principle of Maximal Ageing



Path in space: In Euclidean geometry the curved path has greater length

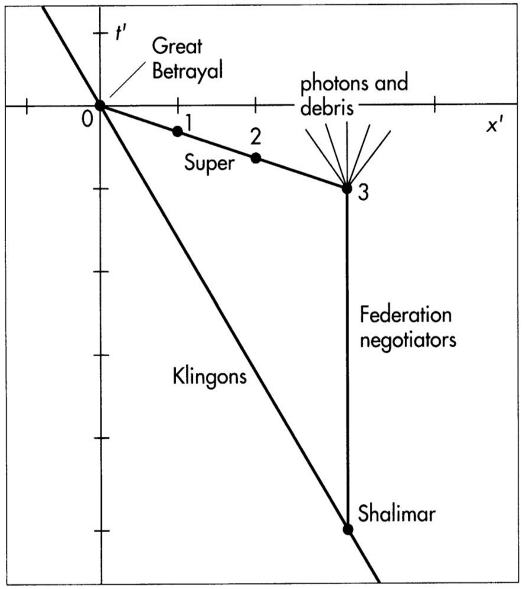
Worldline in spacetime: In Lorentz geometry the curved worldline is traversed in the shorter proper time

Causality – Faster Than Light?



Klingon
("laboratory")
spacetime diagram

The Klingon worldline is the vertical time axis. The Treaty of Shalimar is followed four years later by the Great Betrayal (0) at which Klingons launch the Super, which moves at three times light speed. Travelling from left to right, the Super passes one Federation colony (1) and then another (2). Finally the Super destroys the retreating ship of Federation negotiators (3).



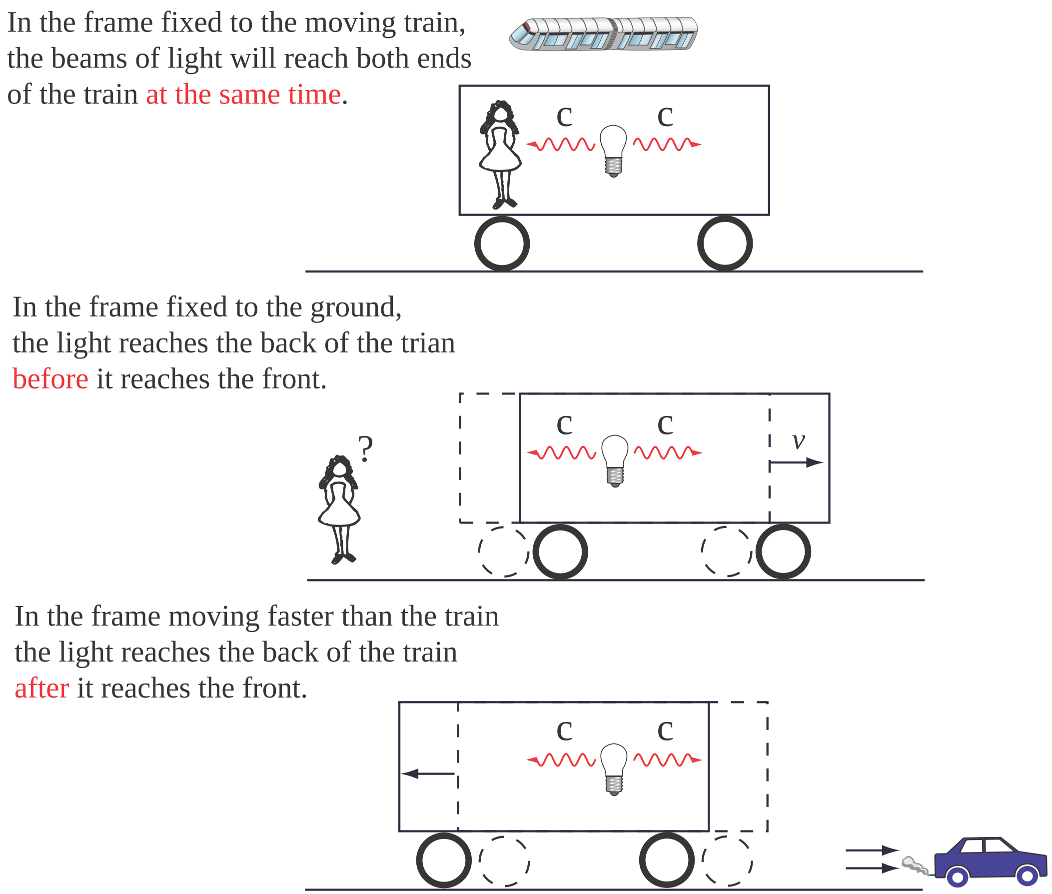
"Rocket" spacetime
diagram of departing
Federation negotiators

In this frame their destruction comes first (3), followed by the passage of the Super from right to left past Federation colonies in reverse order (2 followed by 1). Finally the Super enters the Klingon launcher without doing any further damage (0). Cause and Effect are violated!

Consider a moving train with a light bulb in the middle. If you turn the light bulb on, light will travel both toward the front of the train and also toward the back of the train with speed $c = 3 \times 10^{8}\ ms^{-1}$.

From the point of view of the observer riding on the train, the distances from the light bulb to the front and back ends of the train are the same so *the light will reach both ends <u>at the same time</u>* .

However, from the point of view of the person on the ground, the front of the train is moving away from the light coming toward it while the back of the train is moving closer to the light coming toward it. This means that the distance covered by light going forward will be longer than the light going backwards. And since the *speed of light* is *c* in both directions for the observer on the ground also, *the light will reach the back of the train <u>before</u> it reaches the front of the train*.

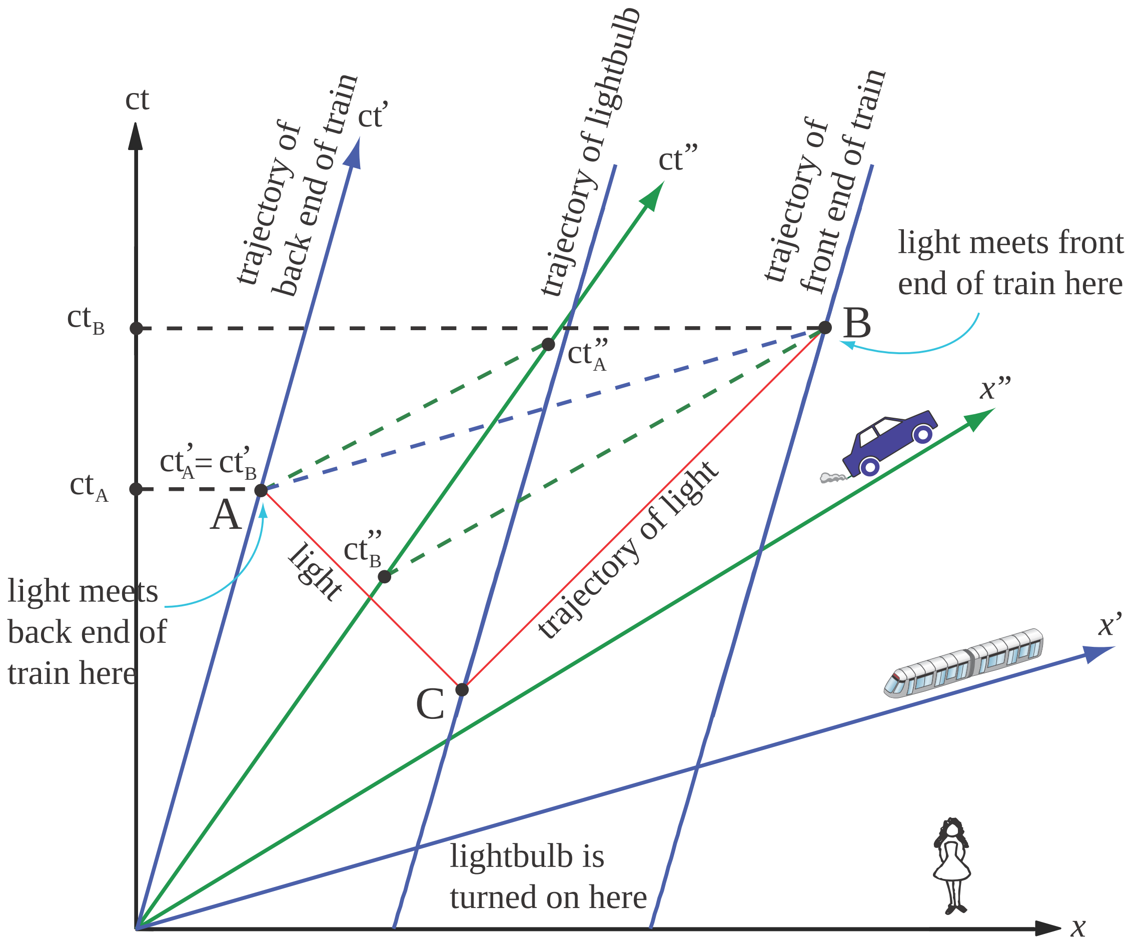


The concept of *before* and *after* actually depends on the observer. Look at the spacetime diagram opposite. Light from the light bulb reaches the back end of the train at **A**, while it reaches the front end of the train at **B**. All observers are observing the same two events **A** and **B**. The spacetime points at which they occur are frame independent.

However, the *chronological order* of the events are frame dependent:

- In the frame fixed to the ground *(x,t)*, **A** happens before **B**.
- In the frame fixed to the train *(x′,t′)*, **A** and **B** happen at the same time.
- In the frame fixed to the sports car *(x″,t″)*, **A** happens after **B**.

This is the unavoidable consequence of the experimental fact that the *speed of light* is the same for all *inertial observers*.

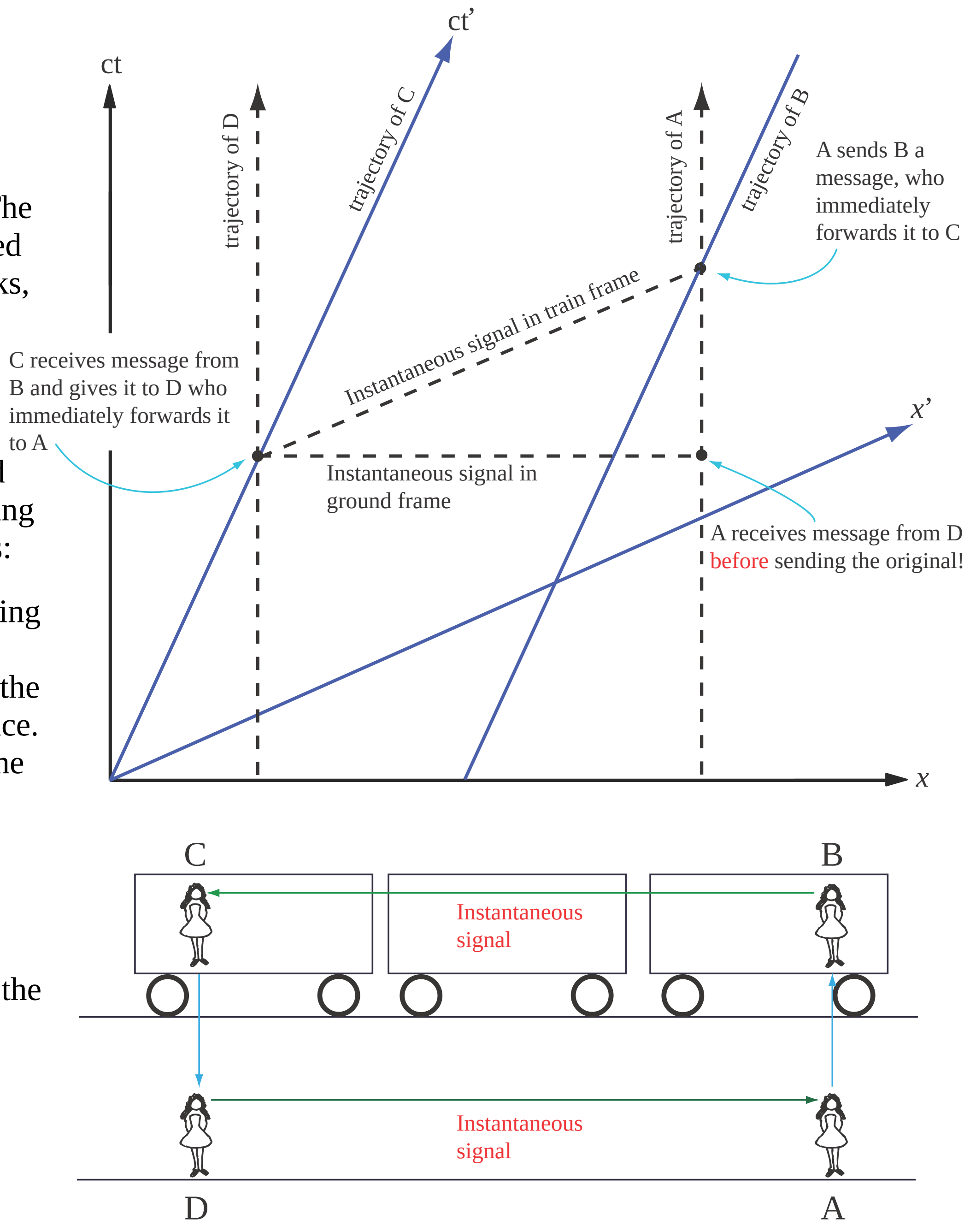


Let's consider another example. This thought experiment is discussed in Chapter 7 of the book "The Einstein Paradox and other Science Mysteries Solved by Sherlock Holmes" by Colin Bruce (Perseus Books, ISBN 0738200239).

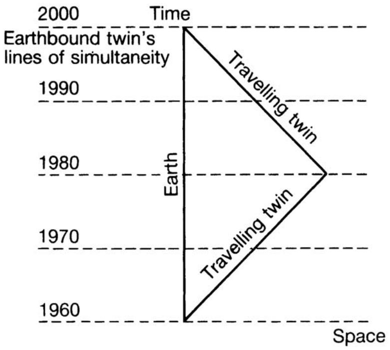
Assume that *instantaneous communication* were possible between remote points. In the spacetime diagram below, A and D are standing by the railroad tracks along which a speeding train passes by carrying B and C. Consider the following sequence of events:

1. A sends a message to B as the leading car carrying B passes by.
2. B relays the message to C riding the last car of the train via the *instantaneous communication* device.
3. C relays the message to D who is standing by the railroad tracks.
4. D relays the message back to A via the *instantaneous communication* device.

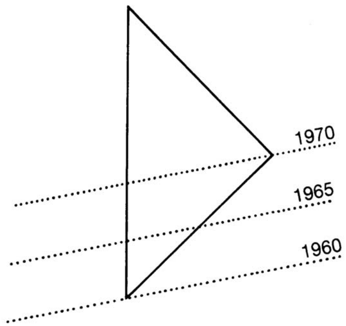
It is clear from the spacetime diagram that A will receive the message from D before she has sent out the original!



The Twins Paradox

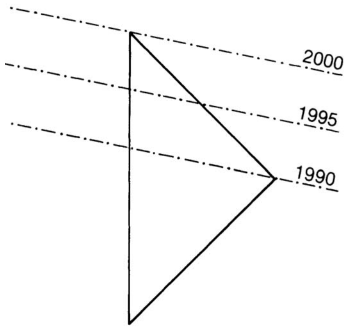


Earth-bound twin's lines of simultaneity

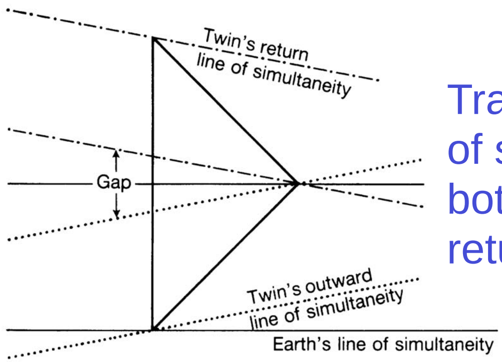


Travelling twin's lines of simultaneity on outward journey

Here $\gamma = 2$

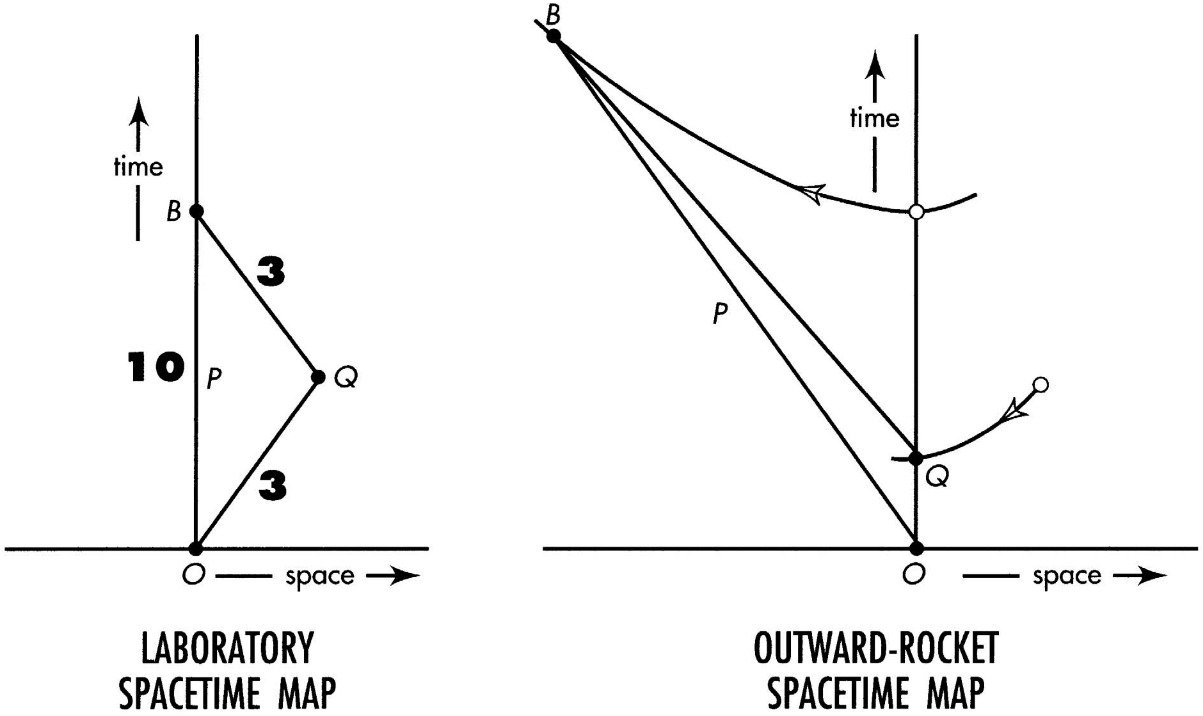


Travelling twin's lines of simultaneity on return journey



Travelling twin's lines of simultaneity on both outward and return journey

The Twins Again



Alternative worldlines (direct OPB and indirect OQB) between events O and B. Note that in the rocket rest frame we need two invariant hyperbolae to show how events Q and B transform. The direct worldline OPB has longer proper time – greater ageing – as computed using measurements from either frame.

Direct world lines show **maximal proper time**

The Speeding Rocket

Initial mass	=	m_i
Final mass	=	m_f
Exhaust speed	=	w
Final speed	=	u

We have:

$$\frac{m_i}{m_f} = \left[\frac{c+u}{c-u}\right]^{c/2w}$$

For w = 10 kms^{-1} we get, for $u = ½\,c$:

$$\frac{m_i}{m_f} = 10^{7157}$$

But for $w \sim c$ we get: $\dfrac{m_i}{m_f} \approx \sqrt{3}$

Is Space Travel Possible?

Suppose you took a trip across the Universe in a spaceship, accelerating all the time at one Earth gravity *g*. How far would you travel in how much time?

See *http://casa.colorado.edu/~ajsh*

Time elapsed on spaceship in years	**Time elapsed on Earth in years**	**Distance travelled in light years**	**To**
0	0	0	Earth (starting point)
1	1.175	.5431	
2	3.627	2.762	
2.337	5.127	4.223	Proxima Centauri
3.962	26.3	25.3	Vega
6.60	368	367	Pleiades
10.9	2.7×10^4	2.7×10^4	Centre of Milky Way
15.4	2.44×10^6	2.44×10^6	Andromeda galaxy
18.4	4.9×10^7	4.9×10^7	Virgo cluster
19.2	1.1×10^8	1.1×10^8	Coma cluster
25.3	5×10^{10}	5×10^{10}	Edge of observable Universe