Complex Numbers I

1. For a) $z_1 = 1 + i, z_2 = -3 + 2i$ and b) $z_1 = 2e^{i\pi/4}, z_2 = e^{-3i\pi/4}$ find
(i) $z_1 + z_2$, (ii) $z_1 - z_2$, (iii) $z_1 z_2$, (iv) z_1/z_2, (v) $|z_1|$, (vi) z_1^*.

2. For $z = x + iy$ find the real and imaginary parts of
(i) $2 + z$; (ii) z^2; (iii) z^*; (iv) $1/z$; (v) $|z|$,
(vi) i^{-5}, (vii) $(1 + i)^2$, (viii) $(2 + 3i)/(1 + 6i)$, (ix) $e^{i\pi/6} - e^{-i\pi/6}$.

3. Find the modulus and argument of each of (i) $R + i\omega L$ (ii) $R + i\omega L + 1/i\omega C$ where R, L, C and ω are all real.

Hence find the modulus and argument of each of (iii) $\frac{V_0 e^{i\omega t}}{R+i\omega L}$ (iv) $\frac{V_0 e^{i\omega t}}{R+i\omega L+1/i\omega C}$ where V_0 is also real. Find also the real and imaginary parts of of (iii) and (iv). (These manipulations are important in a.c. circuit theory, where ω is the angular frequency and $Z = E + i\omega L + 1/i\omega C$ is the complex impedance of a resistance R, inductance L and capicitance C in series.)

4. Change to polar form ($z = re^{i\theta}$)
(i) $-i$, (ii) $\frac{1}{2} - \frac{\sqrt{3}i}{2}$, (iii) $-3 - 4i$, (iv) $1 + i$, (v) $1 - i$, (vi) $(1 + i)/(1 - i)$.

5. Draw in the complex plane
(i) $3 - 2i$, (ii) $4e^{-i\pi/6}$, (iii) $|z - 1| = 1$, (iv) $\Re e(z^2) = 4$, (v) $z - z^* = 5i$,
(vi) $z = te^{it}$ (for real values of the parameter t).

6. Find (i) $(1 + 2i)^7$ (ii) $(1 - 2i)^7/(1 + 2i)^7$

7. Solve for all possible values of the real numbers x and y
(i) $2ix+3 = y-i$, (ii) $(x+2y+3)+i(3x-y-1) = 0$, (iii) $z^2 = z^{*2}$ $(z = x+iy)$,
(iv) $|2x - 1 + iy| = x^2 + iy$.

8. The complex numbers a, b and c represent points in the Argand diagram. Give a geometrical interpretation of $|a - b|$ and $\arg[(a - b)/(a - c)]$.

9. By noting that $e^{in\theta} = (\cos\theta + i \sin\theta)^n$, show that
(a)$\sin 6\theta = 6\cos^5\theta \sin\theta - 20\cos^3\theta \sin^3\theta + 6\cos\theta \sin^5\theta$
(b)$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$.