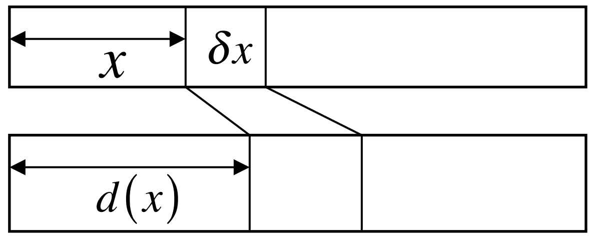
Sound waves

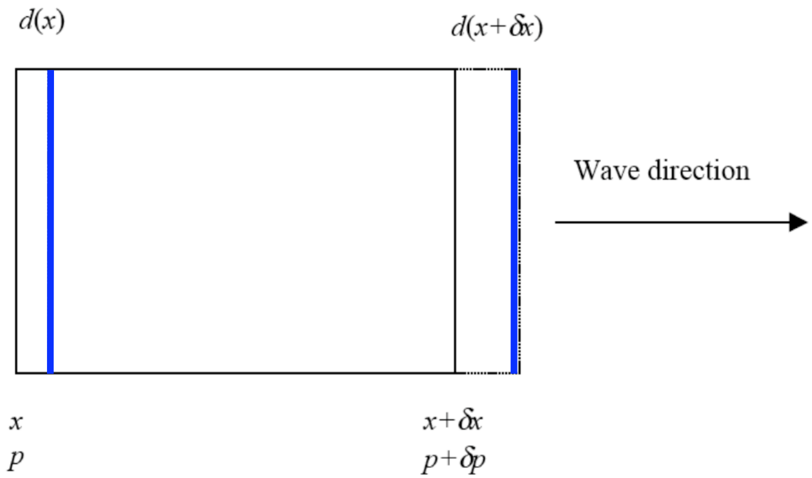
Original configuration



Displacement with sound wave

Longitudinal waves associated with compression of the medium

Area A



$$V_1 = A\delta x$$

No sound wave

$$V_2 = A\big(d(x+\delta x) - d(x)\big) + V_1$$
$$\approx A\frac{\partial d}{\partial x}\delta x + V_1$$
$$= V_1\frac{\partial d}{\partial x} + V_1$$

$$\delta V = V_2 - V_1 = \frac{\partial d}{\partial x}V$$

Pressure

$$P \approx P_0 + \frac{\partial P}{\partial V}\delta V$$

$$= P_0 + \frac{\partial P}{\partial V}\frac{\partial d}{\partial x}V$$

$$= P_0 - K\frac{\partial d}{\partial x}$$

$$\delta V = V_2 - V_1 = \frac{\partial d}{\partial x}V$$

$$K = -V\frac{\partial P}{\partial V}$$ Bulk modulus

Force

$$F = A\left[P(x) - P(x + \delta x)\right]$$

$$= -\frac{\partial P}{\partial x}A\delta x$$

$$= K\frac{\partial^2 d}{\partial x^2}A\delta x$$

$$= K\frac{\partial^2 d}{\partial x^2}V$$

$$= \frac{\partial^2 d}{\partial t^2}\rho V$$

$$\left(P = P_0 - K\frac{\partial d}{\partial x}\right)$$

Newton's 2nd Law

Wave equation

$$\boxed{\frac{\partial^2 d}{\partial x^2} = \frac{\rho}{K}\frac{\partial^2 d}{\partial t^2}}$$

$$\mathrm{v} = \sqrt{\frac{K}{\rho}}$$ velocity

$$\frac{\partial^2 d}{\partial x^2} = \frac{\rho}{K}\frac{\partial^2 d}{\partial t^2}$$

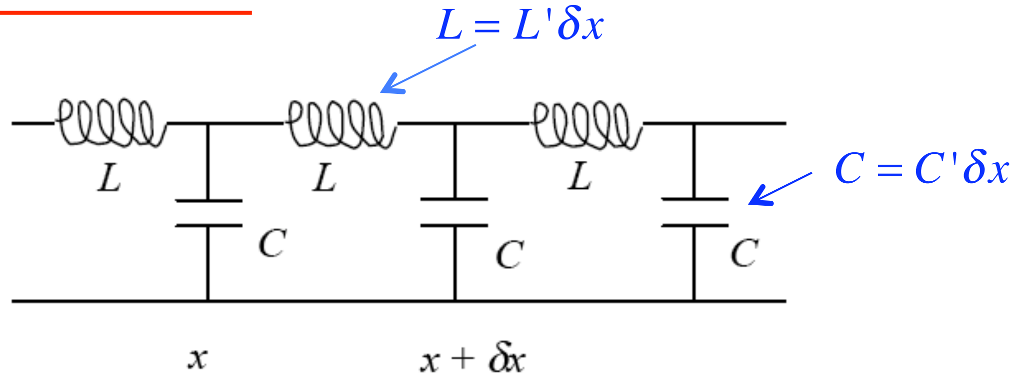
- Isothermal compressions: $PV = \text{constant} \Rightarrow K = -V\frac{\partial P}{\partial V} = P \Rightarrow \text{v} = \sqrt{P/\rho}$
- Adiabatic compressions: $PV^{\gamma} = \text{constant} \Rightarrow K = -V\frac{\partial P}{\partial V} = \gamma P \Rightarrow \text{v} = \sqrt{\gamma P/\rho}$

From kinetic theory we know $P = \frac{1}{3}\rho\overline{\text{v}^2}$ where here v is the molecular speed. Hence $\text{v}_{sound} = \sqrt{\frac{\gamma}{3}\overline{\text{v}^2}}$ and thus $\text{v}_{sound} \approx \text{v}_{rms}$ of molecules since sound is transmitted by moving molecules.

Characteristic impedance $$Z = \frac{K\frac{\partial d}{\partial x}}{\frac{\partial d}{\partial t}} \quad \left(= \frac{P}{\text{v}}\right)$$

Wave in x-direction $$Z = \frac{Kk}{\omega} = \frac{K}{v} = (\rho K)^{1/2}$$

Waves on electrical lines



$$L\frac{\partial I}{\partial t} = -\delta V = -\frac{\partial V}{\partial x}\delta x$$

$$\frac{\partial Q}{\partial t} = C\frac{\partial V}{\partial t} = -\delta I = -\frac{\partial I}{\partial x}\delta x$$

$$\frac{L}{\delta x}\frac{\partial I}{\partial t} = L'\frac{\partial I}{\partial t} = -\frac{\partial V}{\partial x}$$

$$\frac{C}{\delta x}\frac{\partial V}{\partial t} = C'\frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x}$$

$$\frac{\partial^2 V}{\partial t \partial x} = -\frac{1}{C'}\frac{\partial^2 I}{\partial x^2} = -L'\frac{\partial^2 I}{\partial t^2}$$

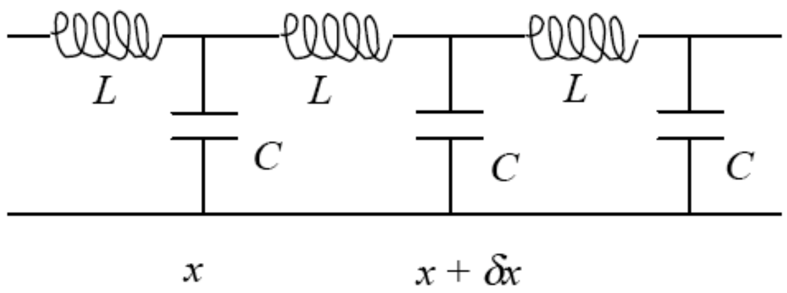
$$\frac{\partial^2 I}{\partial x \partial t} = -C'\frac{\partial^2 V}{\partial t^2} = -\frac{1}{L'}\frac{\partial V}{\partial x^2}$$

$$\boxed{\frac{\partial^2 I}{\partial x^2} = L'C'\frac{\partial^2 I}{\partial t^2}}$$

$$\mathrm{v} = 1/\sqrt{L'C'}$$

$$\boxed{\frac{\partial^2 V}{\partial x^2} = L'C'\frac{\partial^2 V}{\partial t^2}}$$

Waves on electrical lines



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$$\frac{\partial Q}{\partial t} = C\frac{\partial V}{\partial t} = -\delta I = -\frac{\partial I}{\partial x}\delta x$$

$$\frac{C}{\delta x}\frac{\partial V}{\partial t} = C'\frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x}$$

Impedance

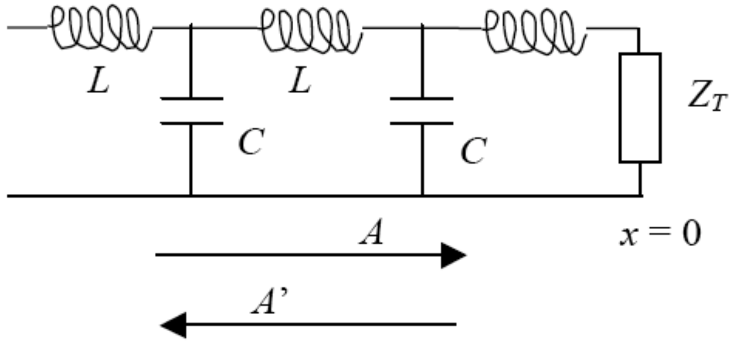
$$V = V_0 \sin(\omega t \mp kx) = IZ_{\mp} \qquad I = (V_0 / Z)\sin(\omega t \mp kx)$$

$$\frac{\partial I}{\partial t} = -\frac{1}{L'}\frac{\partial V}{\partial x} \quad \Rightarrow \quad \frac{V_0}{Z_{\mp}}\omega = \frac{V_0}{L'}k$$

$$Z_{\mp} = \pm\sqrt{L'/C'}$$

$$\mathrm{v} = \frac{\omega}{k} = 1/\sqrt{L'C'}$$

Reflection at a terminated line



$$Z_{\mp} = \pm\sqrt{L' / C'}$$

$$V = A\exp\left(i\left(\omega t - kx\right)\right) + A'\exp\left(i\left(\omega t + kx\right)\right)$$

$$Z_{\_}I = A\exp\left(i\left(\omega t - kx\right)\right) - A'\exp\left(i\left(\omega t + kx\right)\right)$$

$$\frac{V(x=0)}{I(x=0)} = Z_T$$

$$\frac{V}{Z_{\_}I}\Big|_{x=0} = \frac{Z_T}{Z_{\_}} = \frac{A + A'}{A - A'}$$

$$r = \frac{A'}{A} = \frac{Z_T - Z_{\_}}{Z_T + Z_{\_}}$$

Hence

when $Z_T \to 0$, $r \to -1$

when $Z_T = Z_0$, $r = 0$

when $Z_T \to \infty$, $r \to +1$