Standard Model symmetries

$$L_{WD} = \sum_{a=1}^{3} \left(L^{a\dagger} \sigma^{\mu} D_{\mu} L_{a} + e^{ca\dagger} \sigma^{\mu} D_{\mu} e_{a}^{c} + Q^{a\dagger} \sigma^{\mu} D_{\mu} Q_{a} + u^{ca\dagger} \sigma^{\mu} D_{\mu} u_{a}^{c} + d^{ca\dagger} \sigma^{\mu} D_{\mu} d_{a}^{c} \right)$$

$$L_{Yukawa} = i\, \widehat{L}_i\, e_j^c\, H^*\, Y_{ij}^l + i\, \widehat{Q}_i\, d_j^c\, H^*\, Y_{ij}^d + i\, \widehat{Q}_i\, u_j^c\, \tau_2 H\, Y_{ij}^u + c.c.$$

Global symmetries of $L_{WD}: \quad \left(SU(3)_L \times SU(3)_R \times U(1)_L \times U(1)_R \right)^5$

....broken to $\mathrm{U}(1)_{\mathrm{B}} \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ by L_{Yukawa}

Masses and mixing angles

$$L_{Yukawa} = i\,\hat{L}_i\, e^c_j\, H^*\, Y^l_{ij} + i\,\hat{Q}_i\, d^c_j\, H^*\, Y^d_{ij} + i\,\hat{Q}_i\, u^c_j\, \tau_2 H\, Y^u_{ij} + c.c.$$

$$M^l_{ij} = Y^l_{ij} < \overline{H}^0 >$$

$$M^l = U^T_L M^l_{diag} V_R$$

$$L_m = U_L Q, \quad d^c_m = V_R d^c$$

$$L = U^\dagger_L L_m \quad \hat{L} \equiv L^T \sigma_2 = \hat{L}_m \left(U^\dagger_L\right)^T$$

$$i \sum m^l_j \hat{L}_{m,j} e^c_{m,j}$$

$$\hat{L}_i\, Y^l_{ij} e^c_j\, H^* = \hat{L}_m \left(U^\dagger_L\right)^T Y^l V^\dagger_R\, e^c_m H^* = \hat{L}_m\, M^l_{diag} e^c_m H^* / < \overline{H}^0 >$$

Invariant under $U(1)^3$: $\quad L_{m,i} \to e^{i\alpha_i} L_{m,i},\; e^c_{m,i} \to e^{-i\alpha_i} e^c_{m,i}$

i.e. Conserves 3 lepton numbers

Masses and mixing angles

$$L_{Yukawa} = i\,\widehat{L}_i\; e_j^c\; H^*\; Y_{ij}^l + i\,\widehat{Q}_i\; d_j^c\; H^*\; Y_{ij}^d + i\,\widehat{Q}_i\; u_j^c\; \tau_2 H\; Y_{ij}^u + c.c.$$

$$M^d{}_{ij} = Y_{ij}{}^d < \overline{H}^0 > \qquad M^u{}_{ij} = Y_{ij}{}^u < H^0 >$$

$$M^d = U_L^{d,T} M_{diag}^d V_R^d$$

$$Q_m^d = U_L^d Q, \quad d_m^c = V_R^d d^c$$

$$Q = U_L^{d\dagger} Q_m^d \quad \widehat{Q} \equiv Q^T \sigma_2 = \widehat{Q}_m^d \left(U_L^{d\dagger}\right)^T$$

$$\widehat{Q}_i\; d_j^c\; H^*\; Y_{ij}^d = \widehat{Q}_m^d \left(U_L^{d\dagger}\right)^T Y^d V_R^{d\dagger}\; d_m^c H^* = \widehat{Q}_m^d\; M_{diag}^d d_m^c H^* / < \overline{H}^0 >$$

Masses and mixing angles

$$L_{Yukawa} = i\,\widehat{L}_i\, e^c_j\, H^*\, Y^l_{ij} + i\,\widehat{Q}_i\, d^c_j\, H^*\, Y^d_{ij} + i\,\widehat{Q}_i\, u^c_j\, \tau_2 H\, Y^u_{ij} + c.c.$$

$$M^d{}_{ij} = Y_{ij}{}^d < \overline{H}^0 > \qquad M^u{}_{ij} = Y_{ij}{}^u < H^0 >$$

$$M^u = U_L^{u,T} M^u_{diag} V^u_R$$

$$Q^u_m = U^u_L Q, \quad u^c_m = V^u_R u^c$$

$$Q = U_L^{u\,\dagger} Q^{du}_m \qquad \widehat{Q} \equiv Q^T \sigma_2 = \widehat{Q}^u_m \left(U_L^{u\,\dagger}\right)^T$$

$$\widehat{Q}_i\, u^c_j\, \tau_2 H\, Y^u_{ij} = \widehat{Q}^u_m \left(U_L^{u\,\dagger}\right)^T Y^u V_R^{u\,\dagger}\, u^c_m \tau_2 H = \widehat{Q}^u_m\, M^u_{diag} u^c_m H / < H^0 >$$

Invariant under baryon number $U_B(1)$: $\quad B_{m,i} \to e^{i\alpha} B_{m,i}, (u,d)^c_{m,i} \to e^{-i\alpha} (u,d)^c_{m,i}$

Gauge interactions in the mass eigenstate basis

$$Q_m^d = U_L^d Q, \qquad Q_m^u = U_L^u Q$$

Diagonalising up and down masses simultaneously is not a symmetry of L_{WD}

$$L_{WD} \supset j^\mu_{W^\pm} W^\pm_\mu + j^\mu_{W^3} W^3_\mu$$

$$j^\mu_{W^\pm} = d^\dagger_{L,i} \sigma^\mu u_{L,i} \to d^\dagger_m U^d \sigma^\mu U^{u\,\dagger} u_m = d^\dagger_m U^\dagger_{CKM} \sigma^\mu u_m$$

$$j^\mu_{W^3} = \frac{1}{2}\left(u^\dagger_{L,i}\sigma^\mu u_{L,i} - d^\dagger_{L,i}\sigma^\mu d_{L,i}\right) \to \frac{1}{2}\left(u^\dagger_{L,i} U^u \sigma^\mu U^{u\,\dagger} u_{L,i} - d^\dagger_{L,i} U^d \sigma^\mu U^{d\,\dagger} d_{L,i}\right)$$

$$= \frac{1}{2}\left(u^\dagger_{L,i}\sigma^\mu u_{L,i} - d^\dagger_{L,i}\sigma^\mu d_{L,i}\right)$$

The GIM mechanism – no FCNC at tree level

Yukawa couplings in the current basis

$$L_{Yukawa} = i\,\hat{L}_i\, e^c_j\, H^*\, Y^l_{ij} + i\,\hat{Q}_i\, d^c_j\, H^*\, Y^d_{ij} + i\,\hat{Q}_i\, u^c_j\, \tau_2 H\, Y^u_{ij} + c.c.$$

$$M^d{}_{ij} = Y_{ij}{}^d < \overline{H}^0 > \qquad M^u{}_{ij} = Y_{ij}{}^u < H^0 >$$

$$M^u = U_L^{u,T} M^u_{diag} V^u_R$$

$$Q^d_m = U^d_L Q, \quad u^c_m = V^u_R u^c$$

$$Q = U^{d\dagger}_L Q^d_m \qquad \hat{Q} \equiv Q^T \sigma_2 = \hat{Q}^d_m \left(U^{d\dagger}_L\right)^T$$

$$\hat{Q}_i\, u^c_j\, \tau_2 H\, Y^u_{ij} = \hat{Q}^d_m \left(U^{d\dagger}_L\right)^T Y^u V^{u\dagger}_R\, d^c_m \tau_2 H = \hat{Q}^d_m \left(U^{d\dagger}_L\right)^T U^{uT}_L M^u_{diag} u^c_m \tau_2 H / < H^0 >$$

$$L_{Yukawa} \supset \hat{Q}^d_m\, U^T_{CKM} M^u_{diag} u^c_m \tau_2 H / < H^0 > \qquad U_{CKM} = U^u_L U^{d\dagger}_L$$

CKM matrix $U_{CKM} = U_L^u U_L^{d\dagger}$

- $n \times n$ complex matrix $2n^2$ real parameters
- Unitary $U^\dagger U = I$ n^2 constraints ... n^2 real parameters
- $U = P^\dagger U' P'$, P, P' diagonal matrices of phases : $2n-1$ relative phases

  can be absorbed in field redefinition $u_L \to P^\dagger u_L'$, $d_L \to P' d_L'$

$$\Rightarrow n^2 - (2n-1) = (n-1)^2 \text{ parameters}$$

$n \times n$ orthogonal matrix : $\frac{1}{2}n(n-1)$ parameters

$(n-1)^2 - \frac{1}{2}n(n-1) = \frac{1}{2}(n-1)(n-2)$ phases

$n = 3 \Rightarrow 1$ ~~CP~~ phase

$$e.g. \quad U_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

$$= \begin{pmatrix} c_1 & s_1 c_2 & s_1 c_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 c_3 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}$$

NB Many possible conventions possible *c.f.* hep-ph/9912358 Fritzsch & Xing

Wolfenstein parameterisation

$$U_{CKM} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & -\lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad A, \rho, \eta \text{ are of } O(1)$$