The Standard Model (& Beyond)

G. Ross, May 2009

The Standard Model

$$L_{SM} = L_{YM} + L_{WD} + L_{Yu} + L_{H}$$

Strong, weak and EM interactions

A renormalisable, spontaneously broken, local gauge quantum field theory

The Standard Model

$SU(3)\times SU(2)\times U(1)$

$$L_{SM} = L_{YM} + L_{WD} + L_{Yu} + L_H$$

$$L_{YM} = L_{QCD} + L_{I_W} + L_Y$$

$$= -\frac{1}{4g_3^2}\sum_{A=1}^{8} G_{\mu\nu}^A G^{\mu\nu A} - \frac{1}{4g_2^2}\sum_{a=1}^{3} F_{\mu\nu}^a F^{\mu\nu a} - \frac{1}{4g_1^2} B_{\mu\nu}B^{\mu\nu}$$

$$G_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A - f^{ABC} A_\mu^B A_\nu^C, \quad A,B,C = 1,..,8.$$

$$[T^A, T^B] = if^{ABC}T^C$$

$$F_{\mu\nu}^A = \partial_\mu W_\nu^A - \partial_\nu W_\mu^A - f^{ABC} W_\mu^B W_\nu^C, \quad A,B,C = 1,..,3.$$

ε^{ABC}

The Standard Model

$$L_{SM} = L_{YM} + L_{WD} + L_{Yu} + L_H$$

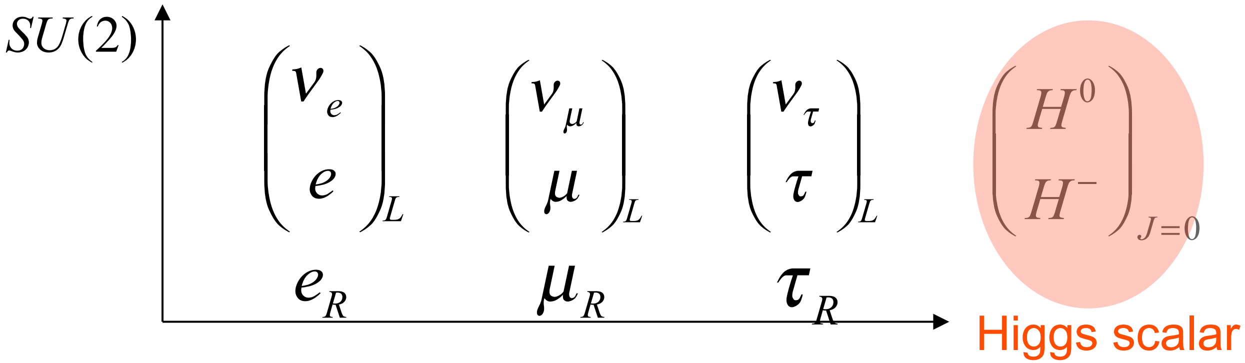
$$L_{YM} = L_{QCD} + L_{I_W} + L_Y$$

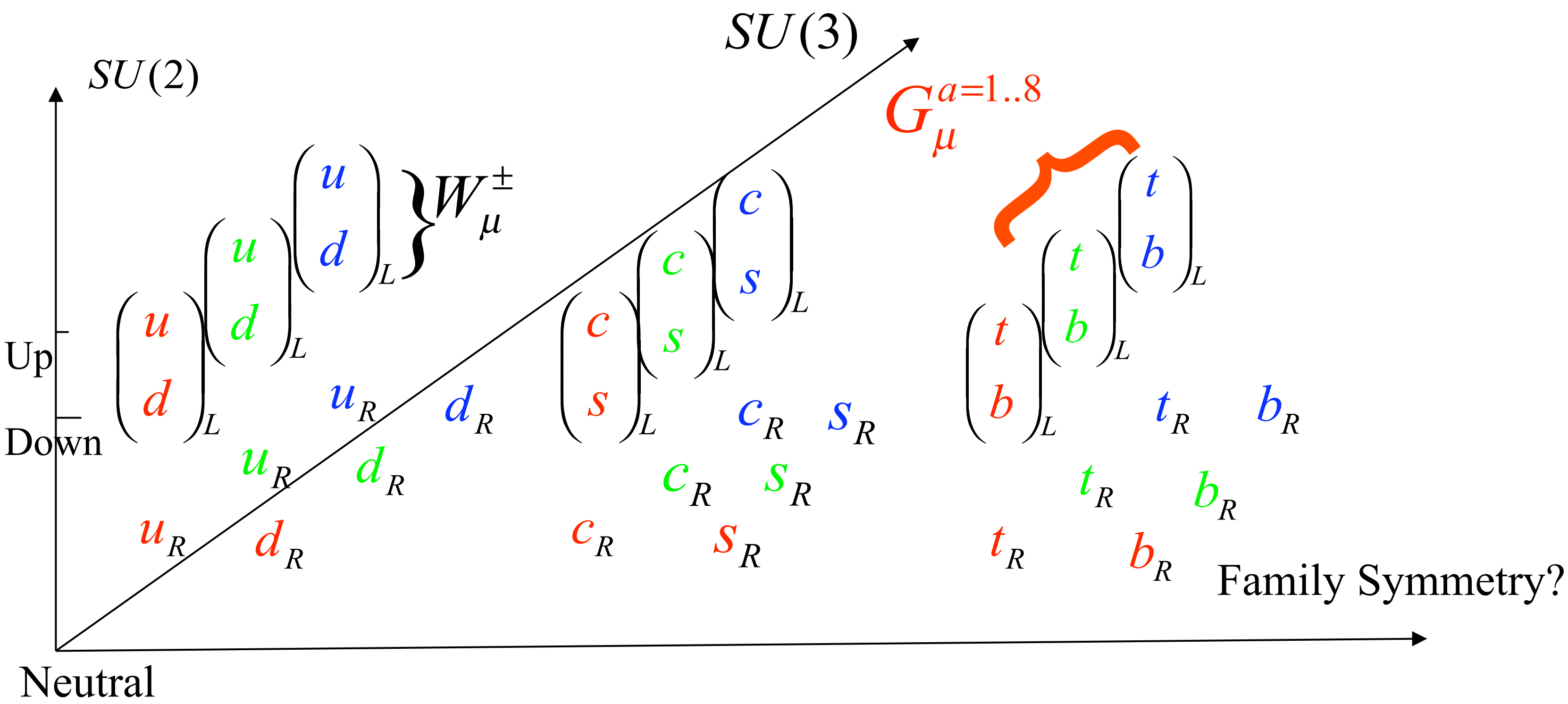
$$= -\frac{1}{4g_3^2}\sum_{A=1}^{8} G_{\mu\nu}^{A} G^{\mu\nu A} + \frac{\theta}{32\pi^2}\varepsilon^{\mu\nu\lambda\sigma}\sum_{A=1}^{8} G_{\mu\nu}^{A} G_{\lambda\sigma}^{A} - \frac{1}{4g_2^2}\sum_{a=1}^{3} F_{\mu\nu}^{a} F^{\mu\nu a} - \frac{1}{4g_1^2} B_{\mu\nu} B^{\mu\nu}$$

$\theta < 10^{-7}$ strong CP problem

Multiplet structure -'chiral'

$SU(3) \otimes SU(2) \otimes U(1)$





$$L_{WD}=\sum_{i=1}^{3}\left(L_i^{\dagger}\sigma^{\mu}D_{\mu}L_i+\bar{e}_i^{\dagger}\sigma^{\mu}D_{\mu}\bar{e}_i+Q_i^{\dagger}\sigma^{\mu}D_{\mu}Q_i+\bar{u}_i^{\dagger}\sigma^{\mu}D_{\mu}\bar{u}_i+\bar{d}_i^{\dagger}\sigma^{\mu}D_{\mu}\bar{d}_i\right)$$

$$D_{\mu}L_i=(\partial_{\mu}+iW_{\mu}+\frac{i}{2}y_1B_{\mu})L_i \qquad y_1=-1$$

$$D_{\mu}\bar{e}_i=(\partial_{\mu}+\frac{i}{2}y_2B_{\mu})\bar{e}_i \qquad y_2=+2$$

$$D_{\mu}Q_i=(\partial_{\mu}+iA_{\mu}+iW_{\mu}+\frac{i}{2}y_3B_{\mu})Q_i \qquad y_3=+\frac{1}{3}$$

?

$$D_{\mu}\bar{u}_i=(\partial_{\mu}-iA_{\mu}+\frac{i}{2}y_4B_{\mu})\bar{u}_i \qquad y_4=-\frac{4}{3}$$

$$D_{\mu}\bar{d}_i=(\partial_{\mu}-iA_{\mu}+\frac{i}{2}y_5B_{\mu})\bar{d}_i \qquad y_5=+\frac{2}{3}$$

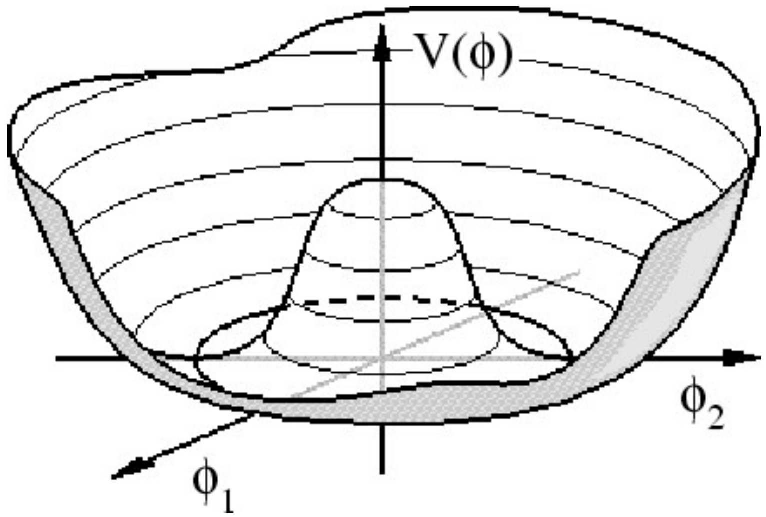
$$L_{Yu}=i\,\hat{L}_i\bar{e}_jH^{*}Y_{ij}^{l}+i\,\hat{Q}_i\bar{d}_jH^{*}Y_{ij}^{d}+i\,\hat{Q}_i\bar{u}_j\tau_2HY_{ij}^{u}+c.c.$$

Spontaneously broken

$$L_H = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - V(H)$$

$$V = -m^2 |H|^2 + \lambda |H|^4$$

$$H = e^{i\xi.\tau} \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$



$$L_H = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} M_W^2 W_\mu^+ W^{-\mu} + \frac{h}{\mathrm{v}}(2 + \frac{h}{\mathrm{v}}) \left(\frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} M_W^2 W_\mu^+ W^{-\mu} \right)$$

The Standard Model (& Beyond?)

Grand Unification

$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

$(\bar{5})_L$:

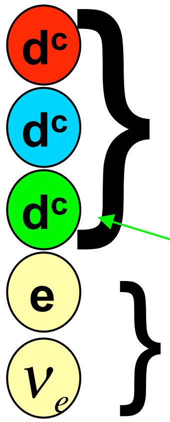
- d^c, d^c, d^c } $SU(3)$
- e, ν_e } $SU(2)$

$$3Q_{d^c} + Q_{e^-} = 0$$

Grand Unification

$SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

$(\bar{5})_L$:



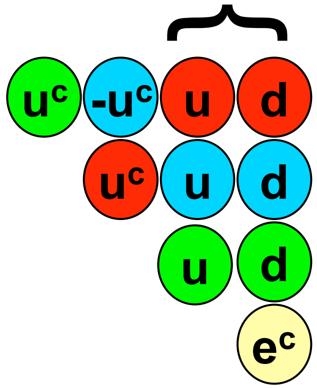
$SU(3)$

$SU(2)$

$$3Q_{d^c} + Q_{e^-} = 0$$

$$Q_{d^c} = 1/3$$

$(10)_L$:



LH states SU(2) doublets

$\nu_{e,L}{}^{c} \equiv \nu_{e,R}$

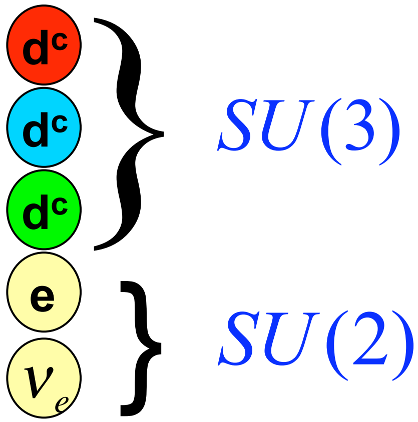
$$(16)_L = (10)_L + (\bar{5})_L + (1)_L$$

Grand Unification

$SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

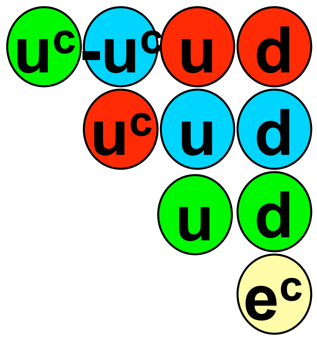
Anomaly free

$(\bar{5})_L$:



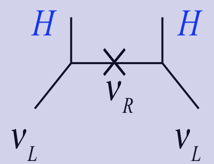
$$3Q_{d^c} + Q_{e^-} = 0$$

$(10)_L$:



$$(16)_L = (10)_L + (\bar{5})_L + (1)_L$$

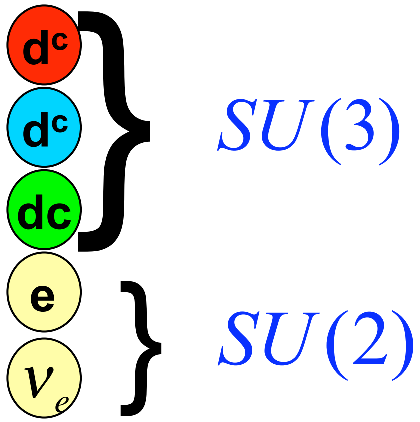
Neutrino masses via seesaw



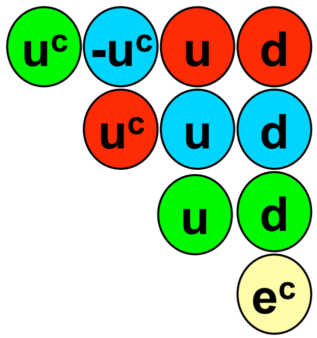
$$M_{\nu_L} = \frac{<H>^2}{M_{\nu_R}}$$

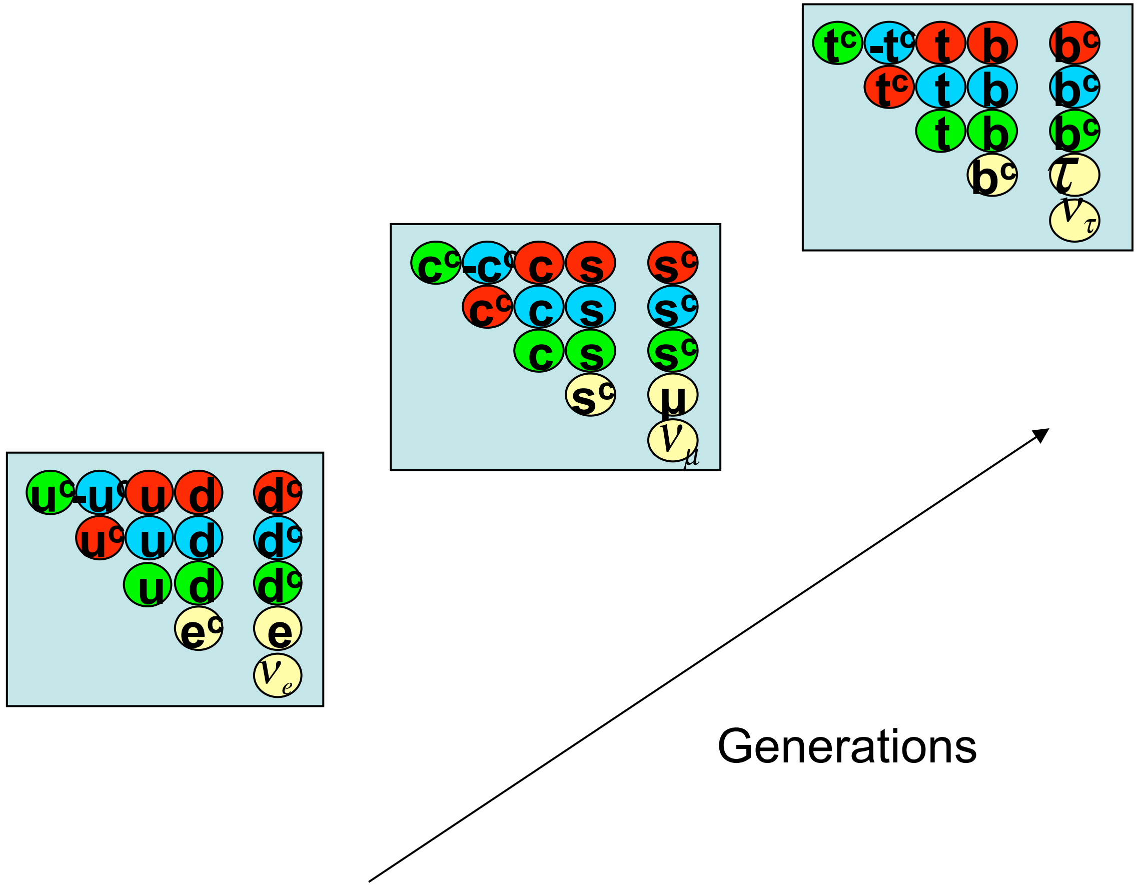
Grand Unification $SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

$(\bar{5})_L$:



$(10)_L$:





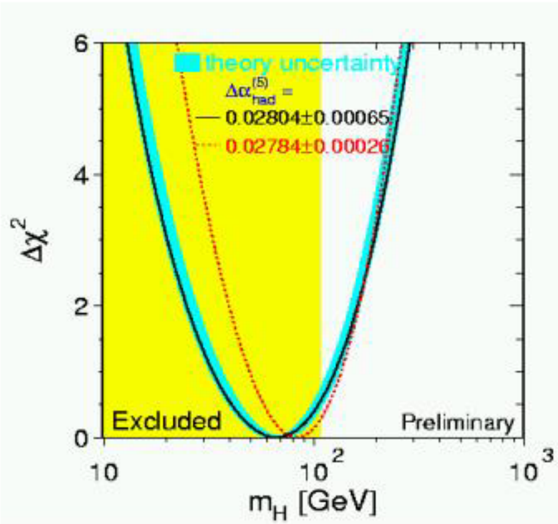
$$(16)_L = (10)_L + (\bar{5})_L + (1)_L$$

Precision tests

New families? :

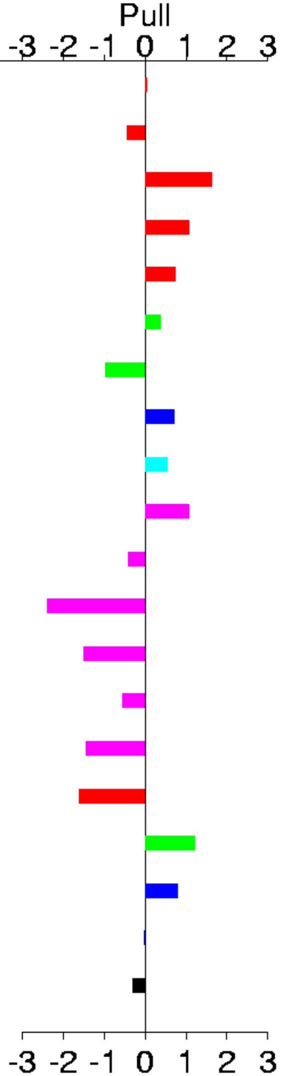
$$N_\nu = 2.984 \pm 0.008$$

Higgs : $H = (H^0 + \overline{H}^0)/\sqrt{2}$ †

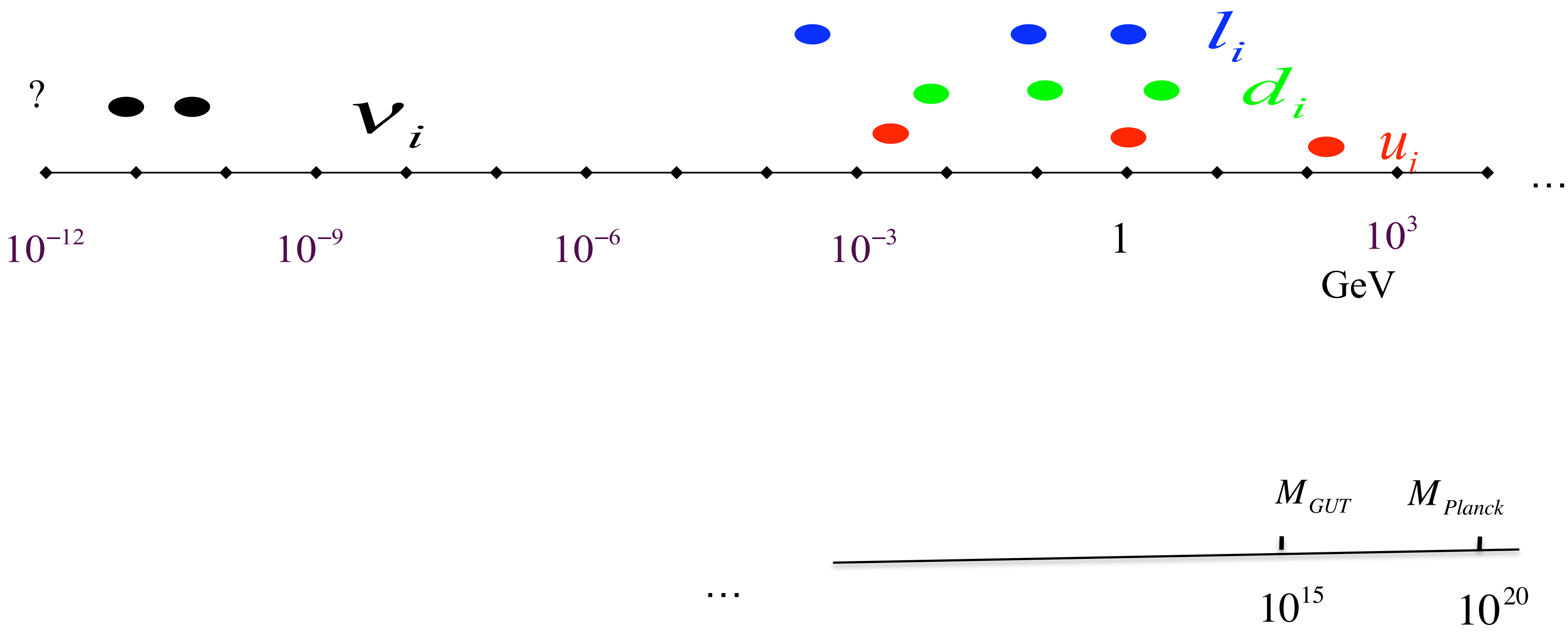


† $\begin{pmatrix} H^0 \\ H^- \end{pmatrix}$ } $W^\pm_\mu$

	Measurement	Pull
m_Z [GeV]	91.1875 ± 0.0021	.05
Γ_Z [GeV]	2.4952 ± 0.0023	-.42
σ^0_{hadr} [nb]	41.540 ± 0.037	1.62
R_l	20.767 ± 0.025	1.07
$A^{0,l}_{fb}$	0.01714 ± 0.00095	.75
A_e	0.1498 ± 0.0048	.38
A_τ	0.1439 ± 0.0042	-.97
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.70
m_W [GeV]	80.427 ± 0.046	.55
R_b	0.21653 ± 0.00069	1.09
R_c	0.1709 ± 0.0034	-.40
$A^{0,b}_{fb}$	0.0990 ± 0.0020	-2.38
$A^{0,c}_{fb}$	0.0689 ± 0.0035	-1.51
A_b	0.922 ± 0.023	-.55
A_c	0.631 ± 0.026	-1.43
$\sin^2\theta^{lept}_{eff}$	0.23098 ± 0.00026	-1.61
$\sin^2\theta_W$	0.2255 ± 0.0021	1.20
m_W [GeV]	80.452 ± 0.062	.81
m_t [GeV]	174.3 ± 5.1	-.01
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02804 ± 0.00065	-.29



Mass scales



The Standard model as an effective field theory…

$SU(3)\times SU(2)\times U(1): G_\mu^{a=1..8}, W_\mu^{a=1..3}, B_\mu$ $\qquad A_\mu \rightarrow A_\mu + \partial_\mu \theta$

$\begin{pmatrix} u \\ d \end{pmatrix}_L, \; u_R, d_R, \; \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \; e_R, \nu_R(?)$ $\qquad f_R \rightarrow e^{i\alpha_R} f_R$

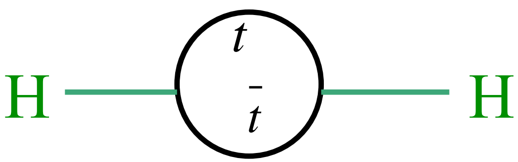
$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

$L^{SM}_{\;\;effective} \supset \cancel{M}_A A_\mu A^\mu + \cancel{m}_f \overline{f_L} f_R \; + M^2_{\;H} |H|^2$

$M_A, m_f \ll M_X, M_{Planck}\,?$

The hierarchy problem :

M_H not forbidden by SM symmetry:



$$M_H^2 \simeq \frac{h_t^2}{16\pi^2}\int_0^{\Lambda^2} dk^2 = \frac{h^2}{16\pi^2}\Lambda^2 \qquad \Lambda \leq 1 TeV\,??$$

The Standard model as an effective field theory…

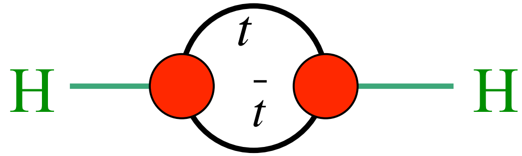
A renormalisable, spontaneously broken, local gauge quantum field theory

- Renormalisable ✔
- Vectors gauge bosons ✔
- Fermions chiral ✔
- Massless gauge bosons - vectorlike couplings ✔
- Massive gauge bosons - chiral couplings ✔
- Spontaneously broken ✗ (hierarchy problem)

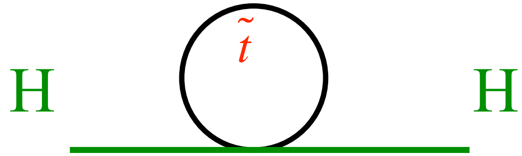
Solutions to the hierarchy problem

$\Lambda \leq 1 TeV$??

- Composite: technicolour, walking technicolor, strongly coupled Standard Model,…

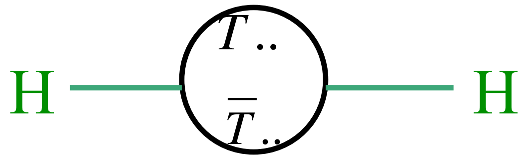


- Symmetry protection — SUSY

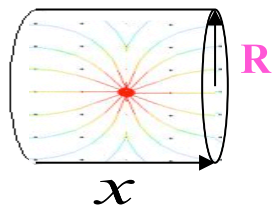


Goldstone: little Higgs

Double protection



- $\Lambda_{fundamental} \simeq 1 TeV$! — Xtra dimensions



$$V(r) = \frac{1}{M_*^{2+d} R^d} \frac{m_1 m_2}{r}, \quad D = 4 + d, \quad r \ll R$$

$$M^2_{Planck} = M_*^2 \left(M_* R \right)^d$$

(• Anthropic) — Split SUSY….