ADVANCED QUANTUM MECHANICS –
(11 lectures)

The Nonrelativistic case – the Schrodinger equation – 6 lectures

- **Introduction to Scattering Theory**

Scattering by a potential
The differential cross section
Stationary states and the scattering amplitude
Calculation of the cross section using probability currents

- **Integral Scattering Equation**

Definition of the Green Function
The Lipmann Schwinger equation
Determination of the Green function (**r** and **k** space)
The Born Series
Calculation of the Born approximation for a Yukawa potential

- **The operator formulation of the Lippmann-Schwinger equation**

Introduction to the operator formalism
The determination of the Green function (regularisation in the complex plane)
The Born Series in operator notation

- **Scattering by a central potential : the method of partial waves**

Angular momentum stationary states
Expansion of a plane wave in terms of free spherical waves
Partial waves in a central potential
The definition of a phase shift
Expression of the cross section in terms of the phase shifts
Unitarity and the optical theorem

Relativistic quantum field theory

Fundamental division of physicist's world :

speed

Action (S)	slow	fast
large	Classical Newton	Classical relativity
small	Classical Quantum mechanics	Quantum Field theory

$\hbar$ $\quad (QM \text{ amplitude} \propto e^{i(\frac{S}{\hbar})})$

c

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The relativistic case - The Klein Gordon equation – 2 lectures

- **The construction of a relativistic wave equation**

Identification of probability density
4-vector formulation
Historical perspective –the problem of the negative energy and probability states,
the Pauli Weisskopf reinterpretation of the probability density
and the Feynman Stuckelberg interpretation of the negative energy states

- **The relativistic treatment of scattering**

The Lorentz invariant form of the electromagnetic potential
The scattering am plitude and the current density
The propagator of the Klein Gordon equation
The determination of the scattering amplitude

The relativistic case – The Dirac equation – 3 lectures

- **The Dirac equation**

Dirac matrices and the relativistic generalisation of the Schrodinger equation
Dirac's derivation – the "square root" of the Schrodinger equation
Hole theory interpretation
Free particle solutions

- **The non -relativistic correspondence**

The introduction of electromagnetism
The coupled equations for the u pper and l ower spinor components
Non-relativistic limit
The gyromagnetic ratio

- **Symmetries**

Angular momentum, spin and helicity
Parity

http://www.physics.ox.ac.uk/users/ross

SCATTERING THEORY AND THE DIRAC EQUATION

- Modern Quantum Mechanics, J.J.Sakurai, Addison-Wesley.

Contains an excellent of the scattering theory topics of the course.

- Quantum Mechanics, C.Cohen-Tanoudgi, B. Diu and F. Laloe, John Wiley & Sons. Vol II

A useful introduction to nonrelativistic scattering theory.

- Quantum Mechanics, L. Schiff, McGraw-Hill.

- Relativistic Quantum Mechanics, I.J.R.Aitchison, Macmillan.

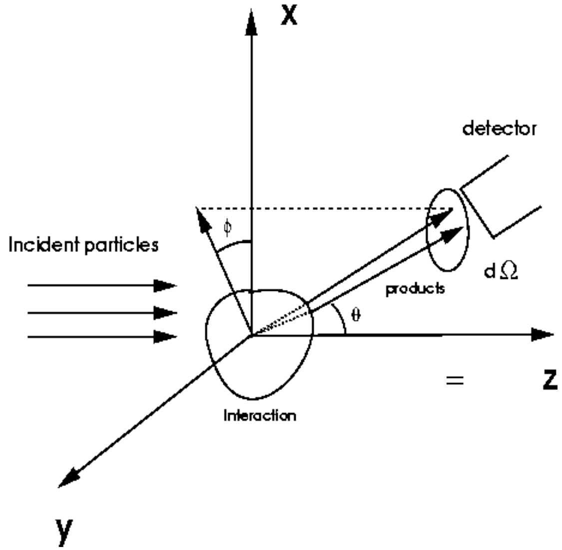
An excellent introduction to the relativistic aspects of the course.

- Relativistic Quantum Mechanics, Bjorken and Drell. McGraw-Hill

A "classic text".

Fundamental experimental objects

$\sigma(a_1 a_2 \rightarrow b_1 b_2 \ldots b_n)$ **Cross section** (Dimension $L^2 = M^{-2}$)



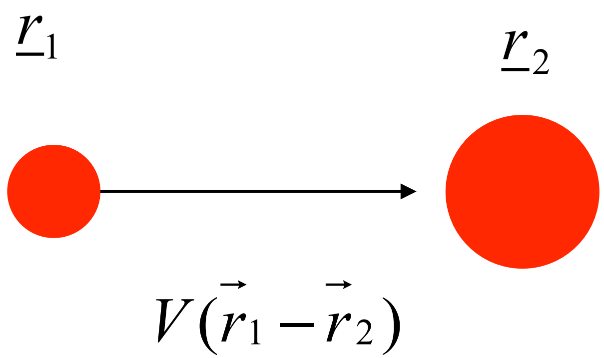
$$dn = F_i \frac{d\sigma}{d\Omega}(\theta,\phi)\, d\Omega \quad (1.1)$$

Differential Cross section $\equiv \dfrac{d\sigma}{d\Omega} = \dfrac{1}{F_i}\dfrac{dn}{d\Omega} \quad \left(\equiv \sigma(\theta,\phi)\right)$

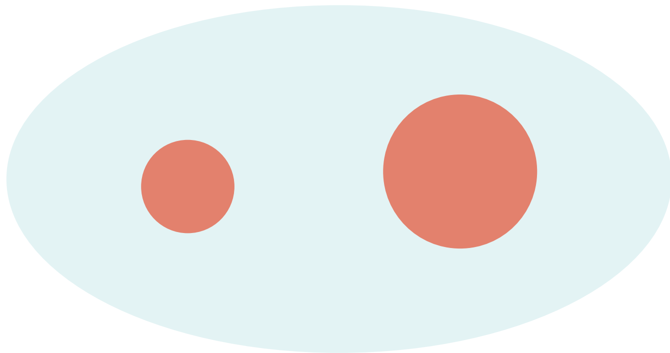
Nonrelativistic wave equation :

Schrodinger equation : $$\left(-\frac{\hbar}{2m}\nabla^2 + V\right)u = Eu = i\hbar\frac{\partial u}{\partial t}$$

K.E.+ P.E.



Potential scattering



(Finite extent assumed here)

Interaction of two particles described by $V(\vec{r}_1 - \vec{r}_2)$

$$\left(-\frac{\hbar}{2m_1}\nabla_1^{\ 2} - \frac{\hbar}{2m_2}\nabla_2^{\ 2} + V(x_1 - x_2, y_2 - y_2, z_1 - z_2)\right)u = E_T u = i\hbar\frac{\partial u}{\partial t}$$

$$x = x_1 - x_2, \ldots \quad \mathrm{MX} = \mathrm{m_1 x_1} + m_2 x_2 \qquad M = m_1 + m_2$$

Schiff p89

$$\Rightarrow \quad \left(-\frac{\hbar^2}{2M}\nabla_X^{\ 2} - \frac{\hbar^2}{2\mu}\nabla_x^{\ 2} + V(x, y, z)\right)u = E_T u$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

Reduced mass

$$u(x, y, z, X, Y, Z) = \psi(x, y, z)U(X, Y, Z)e^{-i(E+E_{CM})t/\hbar}$$

$$\Rightarrow \quad -\frac{\hbar^2}{2M}\nabla_X^{\ 2}U = E_{CM}U \qquad -\frac{\hbar^2}{2\mu}\nabla_x^{\ 2}\psi + \mathrm{V}(x, y, z)\psi = E\psi$$

“Stationary states” – definite energy E

$$i\hbar\frac{\partial\psi}{\partial t}$$

$$-\frac{\hbar^2}{2\mu}\nabla_x^2\psi + \mathrm{V}(x,y,z)\psi = E\psi$$

"Stationary states" – definite energy E

write $E = \frac{\hbar^2 k^2}{2\mu}$

$$-\frac{\hbar^2}{2\mu}\nabla_x^2\psi + V(x,y,z)\psi = E\psi \equiv \frac{\hbar^2 k^2}{2\mu}\psi \quad (1.2)$$

$V(\underline{r}) = \frac{\hbar^2}{2\mu}U(\underline{r})$

$$\left[\nabla^2 + k^2 - U(\underline{r})\right]\psi(\underline{r}) = 0 \quad (1.3)$$

Asymptotic form (V=0 region) of stationary scattering states : scattering amplitude

$$\mathrm{v}_{\mathrm{k}}^{\mathrm{diffractive}}(\underline{r}) \underset{r\to\infty}{\sim} Ae^{ikz} + f_k(\theta,\phi)\frac{e^{ikr}}{r} \quad (1.5)$$

Incident amplitude

Scattering amplitude – want to find this given V

Probability current density

$$-\frac{\hbar^2}{2\mu}\nabla_x^2\psi + V(x,y,z)\psi = i\hbar\frac{\partial\psi}{\partial t} \quad (1.2)$$

$$P(\vec{r},t) = \psi^*(\vec{r},t)\psi(\vec{r},t)$$ Probability density

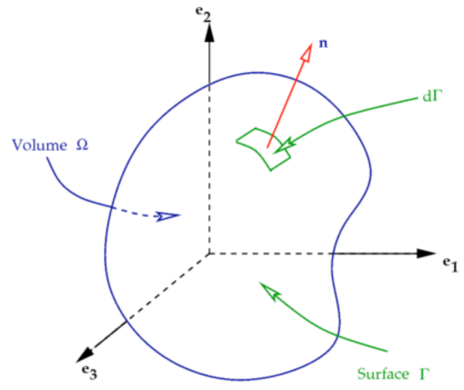
$$\frac{\partial}{\partial t}\int_V P(\vec{r},t)d^3r = \frac{i\hbar}{2\mu}\int_V \nabla.\left(\psi^*\nabla\psi - \left(\nabla\psi^*\right)\psi\right)d^3r$$

$$\vec{J}(\vec{r}) = \frac{-i\hbar}{2\mu}\left(\psi^*\nabla\psi - \left(\nabla\psi^*\right)\psi\right) \quad (1.6)$$

$$\frac{\partial}{\partial t}\int_V P(\vec{r},t)d^3r = -\int_V \nabla.\vec{J}d^3r = -\int_A \vec{J}.\vec{dA}$$

$\vec{J}$ "Probability current density"

$$\Rightarrow \quad \frac{\partial P(\vec{r},t)}{\partial t} + \nabla.\vec{J} = 0$$



Calculation of the cross section

$$\vec{J}(\vec{r}) = \frac{-i\hbar}{2\mu}\left(\psi^{*}\nabla\psi - \left(\nabla\psi^{*}\right)\psi\right) \qquad (1.6)$$

$$J_i = \frac{\hbar k}{\mu} \quad \text{using } \psi \sim Ae^{ikz} + f_k(\theta,\phi)\frac{e^{ikr}}{r} \qquad c.f.(1.5)$$

$$J_d \underset{r\to\infty}{=} \frac{\hbar k}{\mu} \frac{1}{r^2} |\mathrm{f}_\mathrm{k}(\theta,\phi)|^2$$

$$dn = F_i\, \sigma(\theta,\phi)\, d\Omega \qquad (1.1)$$

$$F_i = J_i = \frac{\hbar k}{\mu}$$

$$dn = J_d . d\vec{S} = (J_d)_r r^2 d\Omega = \frac{\hbar k}{\mu} |\mathrm{f}_\mathrm{k}(\theta,\phi)|^2 d\Omega \qquad (1.8)$$

$$\boxed{\sigma(\theta,\phi) \equiv \frac{dn}{d\Omega} = |\mathrm{f}_\mathrm{k}(\theta,\phi)|^2} \qquad (1.9)$$

Units

$$c = 3.10^8 \text{ m/sec}$$

$$\hbar = 10^{-34} \text{ kg m}^2\text{/sec}$$

Natural Units

Length : L
Time : T
Energy : E
or Mass : m

} Choose units such that :

$$c = 1 \quad L/T$$

$$\hbar = 1 \quad E.T \quad (\equiv M.L^2/T)$$

1 unit left : choose $E = 1 \quad GeV$ $(= 10^9 \text{ electron volts} = 1.6\ 10^{-10}\text{J})$