# Feynman rules

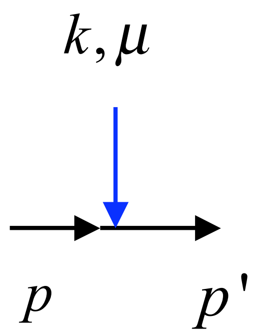
$$iT_{fi} = -i\int d^4y\; f_{p'+}^{+*}(y)\, V(y)\, f_{p+}^{+}(y) = i\int d^4y\; f_{p'+}^{+*}(y)\; ie\left(A^\mu \partial_\mu + \partial_\mu A^\mu\right) f_{p+}^{+}(y)$$

$$f_p^{\pm} = e^{\mp ip.x}\frac{1}{\sqrt{2p^0 V}}$$

$$= -i\int d^4y\; j_\mu^{fi} A^\mu$$

$$j_\mu^{fi} = -ie\left(f_{p'+}^{+*}(y)\left[\partial_\mu f_{p+}^{+}(y)\right] - \left[\partial_\mu f_{p'+}^{*+}(y)\right] f_{p+}^{+}(y)\right)$$

$$= -e(p_f + p_i)_\mu e^{i(p_f - p_i).y}$$

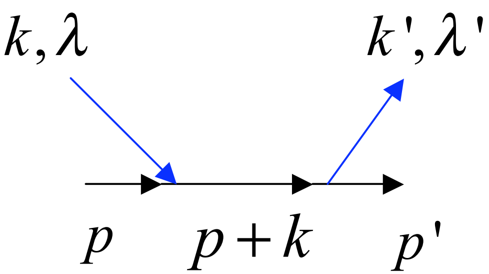


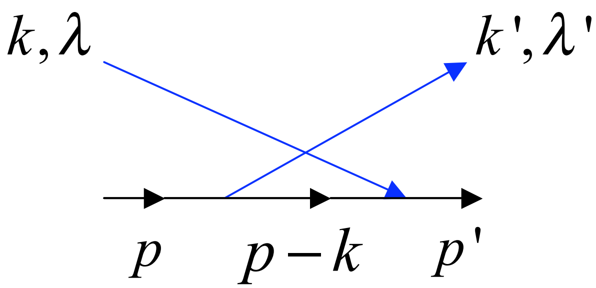

$$ie(p_\mu + p'_\mu)$$

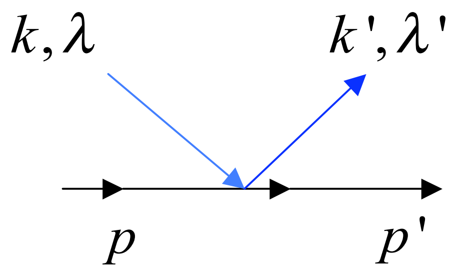
Feynman rule associated with Feynman diagram

## Compton scattering of a π meson

$$\gamma\pi \to \gamma\pi$$



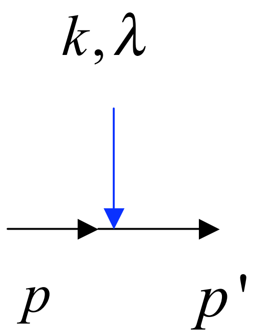



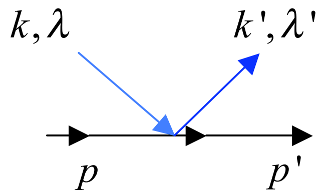

## Feynman rules

Klein Gordon $$(\partial_\mu \partial^\mu + m^2)\psi = -V\psi$$

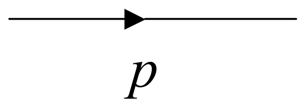
$$V = -ie(\partial_\mu A^\mu + A^\mu \partial_\mu) - e^2 A^2$$




$$ie(p_\lambda + p'_\lambda)$$



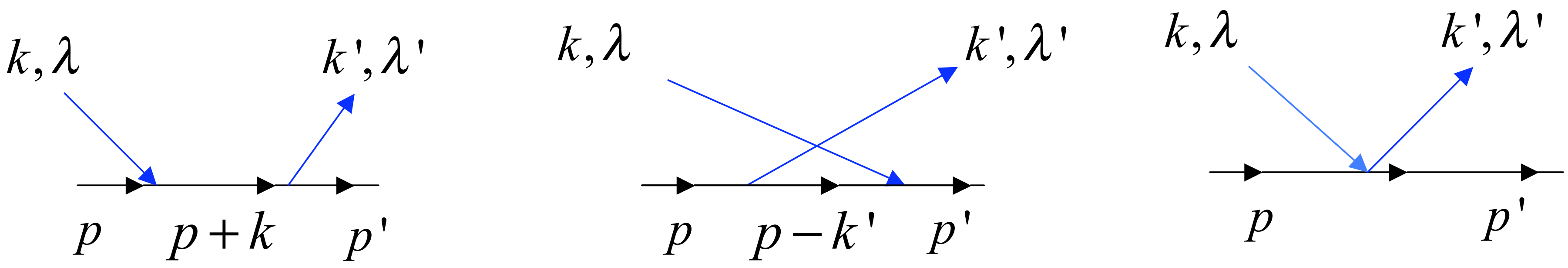

$$ie^2$$




$$\frac{i}{p^2 - m^2}$$

External photon $\varepsilon^\lambda$

## Compton scattering of a π meson

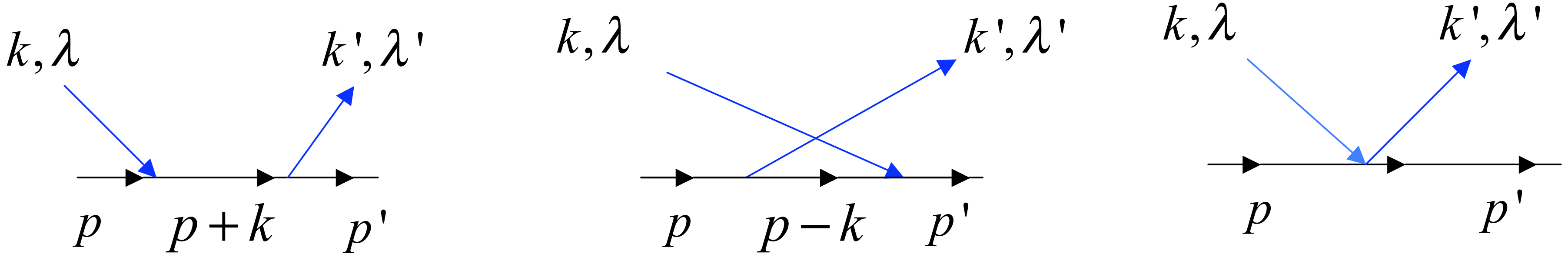



$$i\mathfrak{M}_{fi} = (-ie)^2[\varepsilon.(2p+k)\frac{i}{(p+k)^2 - m^2}\varepsilon'.(2p'+k')$$

$$+\varepsilon.(2p'-k)\frac{i}{(p-k')^2 - m^2}\varepsilon'.(2p-k') - 2i\varepsilon.\varepsilon']$$

$$d\sigma = \frac{V^2}{|\mathbf{v_A}|2E_A 2E_B}\frac{1}{V^4}|\mathfrak{M}|^2\frac{(2\pi)^4}{(2\pi)^6}\delta^4(p_C + p_D - p_A - p_B)\frac{d^3p_C}{2E_C}\frac{d^3p_D}{2E_D}V^2$$

## Compton scattering of a π meson




$$\mathfrak{M}_{fi} = \varepsilon.(2p+k)\frac{i}{(p+k)^2 - m^2}\varepsilon'.(2p'+k')$$

$$+\varepsilon.(2p'-k')\frac{i}{(p+k)^2 - m^2}\varepsilon'.(2p'-k') - 2i\varepsilon.\varepsilon'$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha^2}{m^2}\frac{(\varepsilon.\varepsilon')^2}{\left[1+\frac{k}{m}(1-\cos\theta)\right]^2}$$

$(\varepsilon.p = \varepsilon'.p = 0 \text{ gauge})$

$$\sigma_{total}\,|_{k=0} = \int \frac{d\sigma}{d\Omega}.d\Omega = \frac{8\pi\alpha^2}{3m_\pi^2} \simeq 8.10^{-2}\,GeV^{-2} = 3.10^{-2}\,mb$$

$$\sigma_{total}\,|_{k/m>>1} \simeq \frac{2\pi\alpha^2}{mk}$$

- Dirac's derivation
- The modern view - group representation theory

**The Lorentz group** Rotations $J_i$ Boosts $K_i$

$$[J_i, J_j] = i\varepsilon_{ijk} J_k$$
$$[J_i, K_j] = i\varepsilon_{ijk} K_k$$
$$[K_i, K_j] = -i\varepsilon_{ijk} J_k$$

} Generate the group SO(3,1)

$$\left(M_{\rho\sigma} = i\left(x_\rho \frac{\partial}{\partial x^\sigma} - x_\sigma \frac{\partial}{\partial x^\rho}\right) \qquad J_i = \tfrac{1}{2}\varepsilon_{ijk} M_{jk} \qquad K_i = M_{0i}\right)$$

To construct representations a more convenient (non-Hermitian) basis is

$$N_i = \tfrac{1}{2}(J_i + iK_i)$$

$$[N_i, N_j] = i\varepsilon_{ijk} N_k$$
$$[N_i^\dagger, N_j^\dagger] = i\varepsilon_{ijk} N_k^\dagger$$
$$[N_i, N_j^\dagger] = 0$$

} $SU(2) \otimes SU(2)$ *representation* $(n, m)$

# The Lorentz group

Rotations $J_i$ Boosts $K_i$

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Representations $\quad J_i = N_i + N_i^\dagger$

$(n, m) \quad J = n + m$

| | | |
|---|---|---|
| $(0,0)$ | scalar | $J=0$ |
| $(\tfrac{1}{2},0),\ (0,\tfrac{1}{2})$ | LH and RH spinors | $J=\tfrac{1}{2}$ |
| $(\tfrac{1}{2},\tfrac{1}{2})$ | vector | $J=1$, etc |

## Weyl spinors

$(\tfrac{1}{2},0) \quad (0,\tfrac{1}{2})$

$\psi_L \qquad \psi_R$

2-component spinors of SU(2)

Rotations and Boosts $\quad \psi_{L(R)} \to S_{L(R)}\,\psi_{L(R)}$

$S_{L(R)} = e^{i\frac{\sigma}{2}\cdot\underline{\theta}}$ : Rotations

$S_{L(R)} = e^{\pm\frac{\sigma}{2}\cdot\underline{\nu}}$ : Boosts

†

## Dirac spinor

Can combine $\psi_L, \ \psi_R$ to form a 4-component "Dirac" spinor $\quad \psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$

Lorentz transformations $\quad \psi \to e^{i\omega\sigma}\psi, \qquad \omega\sigma = \omega^{\mu\nu}\sigma_{\mu\nu} = \tfrac{i}{2}\left[\gamma_\mu,\gamma_\nu\right]\omega^{\mu\nu}$

where $\quad \gamma_0 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}, \quad \gamma_i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & o \end{bmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$

Weyl basis

†

$\omega^{0i} \to boosts, \quad \omega^{ij} \to rotations \qquad i,j = 1,2,3$

## Weyl spinors

$(\frac{1}{2},0) \quad (0,\frac{1}{2})$

$\psi_L \qquad \psi_R$

2-component spinors of SU(2)

Rotations and Boosts $\quad \psi_{L(R)} \rightarrow S_{L(R)}\,\psi_{L(R)}$

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## Dirac spinor

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(Dirac gamma matrices, …new 4-vector $\gamma_\mu$ )

$\left\{\gamma^\mu,\gamma^\nu\right\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$

Note : $\quad \psi_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)\psi$

## The Dirac equation

Fermions described by 4-cpt Dirac spinors $\psi$

$$(i\gamma_\mu \partial^\mu - m)\psi = 0$$

New 4-vector $\gamma_\mu$

In momentum space :

$$\psi = e^{-ip.x} u(p)$$

$$\left(\gamma_\mu p^\mu - m\right) u(p) \equiv \left(\not{p} - m\right) u(p) = 0$$