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Key Points:
- Predicting high warming requires understanding feedback temperature dependence
- Feedback temperature dependence can cause a bifurcation under small forcings
- Observational studies likely underestimate the risk of high warming

Supporting Information:
- Supporting Information S1

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Feedback temperature dependence determines the risk of high warming

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Abstract The long-term warming from an anthropogenic increase in atmospheric CO_2 is often assumed to be proportional to the forcing associated with that increase. This paper examines this linear approximation using a zero-dimensional energy balance model with a temperature-dependent feedback, with parameter values drawn from physical arguments and general circulation models. For a positive feedback temperature dependence, warming increases Earth's sensitivity, while greater sensitivity makes Earth warm more. These effects can feed on each other, greatly amplifying warming. As a result, for reasonable values of feedback temperature dependence and preindustrial feedback, Earth can jump to a warmer state under only one or two CO_2 doublings. The linear approximation breaks down in the long tail of high climate sensitivity commonly seen in observational studies. Understanding feedback temperature dependence is therefore essential for inferring the risk of high warming from modern observations. Studies that assume linearity likely underestimate the risk of high warming.

1. Introduction

Studies of equilibrium climate sensitivity often assume that the long-term warming caused by increasing atmospheric CO_2 to some new fixed level would be proportional to the radiative forcing associated with that CO_2 increase [e.g., *Andrews et al.*, 2012; *Otto et al.*, 2013]. This is equivalent to assuming that the feedbacks that regulate the planet's surface temperature maintain the same strength under anthropogenic forcings (e.g., a few doublings of CO_2). This allows the general response to be characterized by the equilibrium climate sensitivity (ΔT_{2x}), the warming caused by doubling CO_2 from preindustrial levels.

This linear assumption is always made with the caveat that for large enough forcings, feedbacks do change strength, often as a result of changes in temperature [e.g., *Hansen et al.*, 1984], making the equilibrium warming response nonlinear with forcing. An extreme case is given by the runaway greenhouse [e.g., *Nakajima et al.*, 1992], in which as the world warms, the water vapor feedback strengthens, eventually causing the climate to jump to a warmer state. A warmer world also has a weaker surface albedo feedback, due to the disappearance of snow and ice [*Wetherald and Manabe*, 1975]. We refer to these types of nonlinearity as "equilibrium nonlinearity" to differentiate them from "transient nonlinearity" [e.g., *Armour et al.*, 2013], in which the emergence of temporary feedbacks causes the apparent sensitivity to differ from the long-term response [e.g., *Senior and Mitchell*, 2000].

Studies have explored the importance of equilibrium nonlinearity by running general circulation models (GCMs) to equilibrium under different levels of CO_2 forcing [e.g., *Manabe and Bryan*, 1985; *Colman et al.*, 1997; *Caballero and Huber*, 2013]. While many models do not show much difference in sensitivity under different amounts of CO_2 forcing, some do. For example, the atmospheric component of the Max Planck Institute Earth System Model, version 6 (ECHAM6) warms about as much from its third doubling of CO_2 as from its first and second doubling combined, with each doubling having roughly the same forcing [*Meraner et al.*, 2013]. This invites the question of under what conditions the linear approximation breaks down.

We assess the limits of equilibrium linearity by exploring the behavior of a zero-dimensional energy balance model with a term representing feedback temperature dependence, following *Roe and Baker* [2007] and *Zaliapin and Ghil* [2010]. We estimate a reasonable range of values for this dependence by diagnosing it from GCM experiments and by using physical arguments. We find that a positive feedback temperature dependence can cause Earth's sensitivity to increase substantially from its linear approximation, with some parameter choices leading to a jump to a warmer state under only one or two doublings of CO_2.

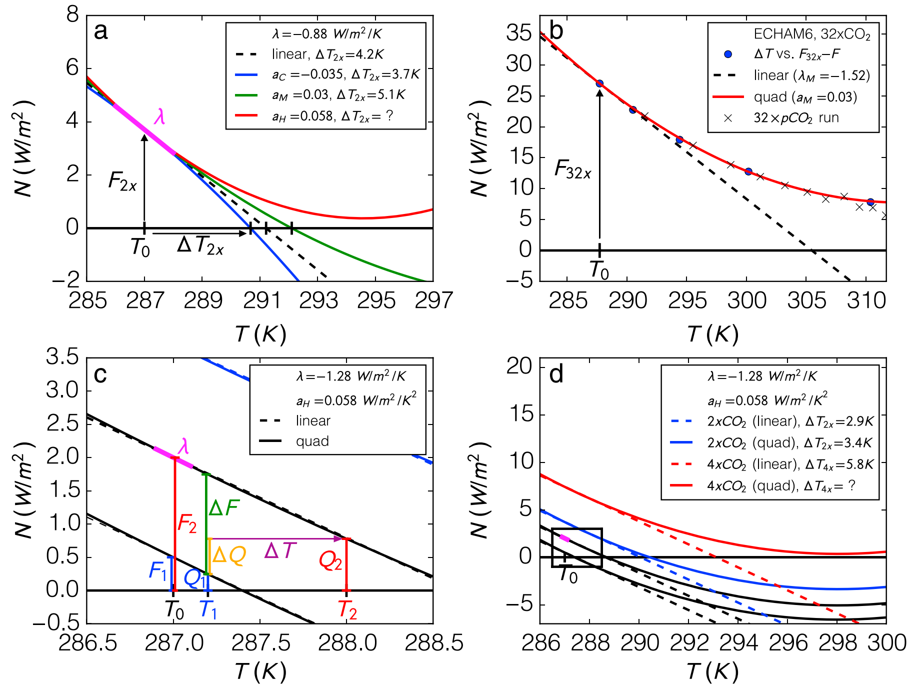

Figure 1. Global annual mean net top-of-atmosphere energy flux N (downward positive) as a function of global annual mean surface temperature T, where each curve has a fixed CO_2 concentration. (a) Doubling CO_2 increases N by a forcing $F_{2x} \approx 3.71$ W/m^2 [*Collins et al.*, 2013]. All four curves have the same preindustrial feedback, λ (the slope of N at T_0), but different values for the feedback temperature dependence, a, resulting in different warming responses ΔT_{2x}. Colored lines have values of a where the subscripts correspond to GCMs in Figure 2. For a_H (red curve), the feedback becomes positive, causing runaway warming. (b) ECHAM6 under five CO_2 doublings. We quadratically fit N to preindustrial conditions and the equilibrium response to four CO_2 forcings (blue circles [*Meraner et al.*, 2013]), adding F_{32x} to estimate $N(T, 32C_0)$ (red curve). The time series of T versus N from the $32xCO_2$ run (black crosses) follows this curve, until the model "blows up" (data from T. Mauritsen, personal communication, 2015). (c) Illustrative scenario with fixed values λ and a, showing how λ can be estimated from observations of surface temperature (T), forcing (F), and heat uptake (Q) from times 1 and 2. Warming appears linear. (d) Same scenario after one (blue) and two (red) CO_2 doublings, with the previous panel inset. The linear approximation works for one doubling, but under two doublings, the quadratic model runs away.

In particular, the linear approximation breaks down for the long tail of high-sensitivity cases, which play a large role in assessments of economic risk [*Weitzman*, 2011]. We show that observational estimates of climate sensitivity likely underestimate the risk of high warming by neglecting equilibrium nonlinearity, and that understanding this nonlinearity is essential for reducing our uncertainty of these high-risk scenarios.

2. Model Setup and Methods

We use a zero-dimensional energy balance model of equilibrium climate sensitivity (Figure 1). Let T, C, and N be the global annual means of surface temperature, atmospheric CO_2 concentration, and net top-of-atmosphere energy flux, respectively. For N, positive values represent a net downward flux. N can be expressed as a function of T and C (see supporting information, Section S1), and in turn, N acts to warm or cool the surface ($dT/dt \propto N$).

Suppose that the preindustrial Earth was in equilibrium (i.e., $N(T_0, C_0) = 0$, where $T_0 \approx 287$ K and $C_0 \approx 270$ ppm are the preindustrial values of T and C, respectively). If the CO_2 concentration is increased, N would increase by a radiative forcing F, causing the planet to gain energy and T to increase until N is 0 again, resulting in an equilibrium warming ΔT.

The slope of N at T_0 with respect to T is the preindustrial feedback, which we call λ (i.e., $\lambda \equiv \frac{\partial N}{\partial T}|_{T_0,C}$). We use "feedback" to describe the sum of *all* the ways T changes N, including the Planck effect. In our sign convention, a negative λ implies a stable preindustrial Earth, and a more negative λ implies a less sensitive

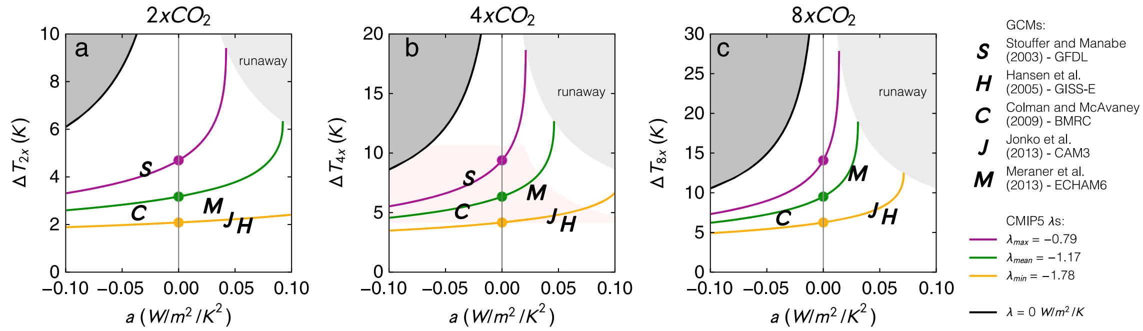

Figure 2. The feedback temperature dependence a versus (a) ΔT_{2x}, (b) ΔT_{4x}, and (c) ΔT_{8x} (the equilibrium warming from one, two, and three doublings of CO_2, respectively). Letters represent GCMs and are centered on pairs of a (quadratic estimate, see supporting information, Section S3) and ΔT_{nx} (reported values). Colored lines show how quadratic estimates of ΔT_{nx} (using equation (1)) vary with a for fixed values of λ, assuming forcings of $F_{nx} = log_2(n)3.71$ W/m^2. Colored circles where these lines intersect the center vertical give linear estimates of ΔT_{nx}. Some lines intersect the shaded regions in the upper right corner of each panel. For values of a greater than these intersections, the quadratic model experiences runaway warming. If we assume that the preindustrial climate was stable, then $\lambda \leq 0$, and the dark region in the upper left-hand corner of each panel is inaccessible under the quadratic model. CMIP5 models must lie in the pink region in Figure 2b to avoid quadratic runaway under RCP8.5. Note that the y axis increases proportionally in each panel [*Colman and McAvaney*, 2009; *Hansen et al.*, 2005; *Jonko et al.*, 2013; *Meraner et al.*, 2013; *Stouffer and Manabe*, 2003].

preindustrial Earth. If N is a linear function of T (Figure 1a, dashed black line), this feedback remains the same under warming, and we get the linear estimate $\Delta T = -F/\lambda$.

We can explore the accuracy of this linear estimate by adding a quadratic term representing the temperature dependence of the feedback [e.g., *Roe and Baker*, 2007; *Zaliapin and Ghil*, 2010]. We call the coefficient of this term a (i.e., $a \equiv \frac{1}{2}\frac{\partial^2 N}{\partial T^2}|_{T_0,C}$). There is also a quadratic term representing the CO_2 dependence of the feedback. While this term can influence the exact value of ΔT, it does not have the same potentially extreme effect on ΔT as the feedback temperature dependence (see supporting information, Section S2 and Figure S3). To clearly explain this latter effect, we follow these earlier studies in leaving out feedback CO_2 dependence.

To get the quadratic estimate of ΔT for a given forcing F, we solve the quadratic equation

$$-F = \lambda \Delta T + a \Delta T^2. \quad (1)$$

Negative a (Figure 1a, blue line) implies that the feedback gets more negative under warming, giving less than linear warming. Positive a (Figure 1a, green and red lines) implies that the feedback gets less negative under warming, giving greater than linear warming. For large enough a (Figure 1a, red line), the feedback becomes positive before equilibrium is reached, and the quadratic model warms indefinitely. If we include still higher-order nonlinear terms of N, these terms would ensure that indefinite warming would not occur (e.g., supporting information, Figures S1 and S2).

To explore the applicability of the linear approximation, we compare linear and quadratic estimates of ΔT_{2x}, ΔT_{4x}, and ΔT_{8x} for reasonable values of λ and a. We estimate the range of reasonable λ by dividing the minimum and maximum ΔT_{4x} from the Coupled Model Intercomparison Project Phase 5 (CMIP5) *abrupt4xCO$_2$* experiment (4.16 and 9.34 K [*Andrews et al.*, 2012]) by $-F_{4x} \approx -7.42$ W/m^2 [*Collins et al.*, 2013], giving $-1.78 \leq \lambda \leq -0.79$ W/m^2/K.

We can estimate the range of likely a by estimating a for GCMs that were run to equilibrium under different CO_2 forcings, using quadratic regression (see supporting information, Section S3; Section S2 also discusses the effect of CO_2 dependence on these estimates). Estimates of a for various GCMs are given by the letters in Figure 2 and suggest a range of $-0.04 \leq a \leq 0.06$ W/m^2/K^2. This range is similar to an earlier survey, ($-0.06 \leq a \leq 0.06$ W/m^2/K^2 [*Roe and Armour*, 2011]). Our range is narrower because we only include studies that use CO_2 forcings, as opposed to solar forcings or conditions at the Last Glacial Maximum.

As an example, Figure 1b shows a quadratic fit of N (red line) to equilibrium runs of ECHAM6 (blue dots [*Meraner et al.*, 2013]), where the blue dots represent preindustrial conditions and the response to four successive doublings of CO_2. We have added a forcing F_{32x} to predict ECHAM6's response to five CO_2 doublings. Instantaneous values of T versus N for the 32xCO_2 run closely track our curve (black crosses, T. Mauritsen, personal communication, 2015). The quadratic model successfully predicts that the five doubling run of ECHAM6 does not equilibrate, although the exact reasons for this blowup are unclear.

To understand and justify this range of feedback temperature dependence, we look at the various physical processes that contribute to it. The Planck feedback gets more negative under warming, contributing a term of $\frac{1}{2}(d^2[\sigma T^4]/dT^2|_{255\text{K}}) \approx -0.02$ W/m^2/K^2, where 255 K is Earth's equilibrium temperature. The water vapor feedback gets more positive, possibly offset by changes to the lapse rate feedback [*Colman et al.*, 1997]. *Meraner et al.* [2013] studied the changing sensitivity of a moist adiabat with fixed relative humidity and tropopause temperature, representing the combined water vapor, lapse rate, and Planck feedbacks. We estimate $a \approx 0.05$ W/m^2/K^2 for their model. The surface albedo feedback weakens under warming as snow and ice disappear. An early study of this weakening with idealized geography [*Manabe and Bryan*, 1985] found $a \approx -0.1$ W/m^2/K^2 for the combined noncloud feedbacks. The contributions to a from cloud feedbacks and cross terms between feedbacks are uncertain, given our uncertainty about the cloud feedback itself.

3. Results

The colored lines in Figure 2 show how quadratic estimates of ΔT_{2x}, ΔT_{4x}, and ΔT_{8x} vary with a for fixed values of λ. Linear estimates are given by the circles where these lines intersect the center vertical. When a is positive, warming makes the feedback become less negative. In turn, a less negative feedback causes additional warming. These two effects feed on each other so that positive values of a can greatly increase warming from the linear estimate. This effect gets stronger for less negative λ and for larger CO_2 forcing.

Some colored lines intersect the shaded region in the upper right corner of each subfigure. For values of a greater than these intersection points, these values of λ make the quadratic model warm indefinitely, like the red curves in Figures 1a and 1d. In these cases, the stable equilibrium that the quadratic model is tracking disappears in a saddle-node bifurcation [*Strogatz*, 1994] (see supporting information, Figure S2). If Earth has values of λ and a such that for a given forcing F, the quadratic model warms indefinitely, we say that Earth experiences a "quadratic runaway" for that forcing. If Earth has $\lambda_{\text{max}} = -0.79$ W/m^2/K and $a_J = 0.042$ W/m^2/K^2 [*Jonko et al.*, 2013], it would experience a quadratic runaway from just one CO_2 doubling. Under two doublings, $\lambda_{\text{mean}} = -1.17$ W/m^2/K and $a_H = 0.058$ W/m^2/K^2 [*Hansen et al.*, 2005] would lead to a quadratic runaway. This latter case appears linear under only one doubling, with a ΔT_{2x} of only 3.4 K (Figure 1d).

If Earth experiences a quadratic runaway for a forcing F, the warming response to that forcing cannot be estimated from the quadratic model, since the quadratic model warms indefinitely, which is clearly unphysical. To know the warming caused by F, we would have to include higher-order terms, and the exact value of the resulting warming would depend sensitively on the value of these higher-order terms (see supporting information, Figure S1b). Since we only have detailed observations of Earth over a relatively small temperature range, our physical (GCM) modeling of these higher-order terms is poorly constrained. Further, as Earth warms, new feedbacks such as those from the melting of ice sheets, changes to the ocean circulation, and the release of soil carbon come into play [*Lunt et al.*, 2010], making our estimation of these higher-order terms still more difficult.

As a result, if Earth quadratically runs away due to a forcing F, the resulting warming may be inherently uncertain. In fact, if these higher-order terms are negative enough, Earth might not run away at all, even though its quadratic approximation runs away (supporting information, Figure S1, green curve). However, if the higher-order terms are not very negative, Earth will go through a period of runaway warming before ultimately stabilizing (e.g., supporting information, Figures S1 (red and blue curve) and S2). These jumps to warmer states can be hundreds of degrees, as in the case of the runaway greenhouse, or much fewer, depending on the higher-order terms discussed. Jumps to warmer states far short of the runaway greenhouse have been seen in lower dimensional models [*Abbot and Tziperman*, 2008; *Popp et al.*, 2015] and GCMs [*Popp*, 2014] and may explain Cenozoic hothouse climates [*Pierrehumbert*, 2013].

The quadratic model breaks down in the limit of high warming and large higher-order temperature dependence, just like the linear model. The quadratic runaway is simply a case where this breakdown is guaranteed. Care should be taken in using the quadratic model to estimate large warmings. However, a main goal of this paper is to estimate the regions of parameter space for which linearity breaks down. Higher-order terms only grow these regions. Further, the quadratic approximation often works well (e.g., supporting information, Figure S1a). Even if the quadratic model does run away, the forcing and temperature at which the quadratic estimate of N reaches its minimum can predict when Earth will begin a potential jump to a warmer state (e.g., Figure 1b and supporting information, Figure S2e) and serves as a rough lower bound on warming for these quadratic runaway cases.

Jumps to warmer states and other large deviations from equilibrium linearity under only one or two doublings are uncommon in the published literature, outside of occasionally appearing in perturbed physics ensembles [e.g., *Tomassini et al.*, 2014]. When such deviations do appear, they are usually in response to much larger forcings [e.g., *Boer et al.*, 2005]. Of the five GCMs analyzed here, all but ECHAM6 occupy regions where the linear approximation works well, having either highly negative preindustrial feedbacks (λ), negative or small feedback temperature dependence (a), or both. There are two main reasons why equilibrium nonlinearity may be uncommon in the published literature.

First, GCMs that experience quadratic runaway would behave in ways suggestive of a nonphysical loss of stability, i.e., model "blow up." In a perturbed physics ensemble with the MIROC3.2 GCM [*Yokohata et al.*, 2010], 20% of parameter sets created GCMs that after being subjected to an abrupt doubling of CO_2, did not regain stability after 70 years. As a result, these cases were disregarded. A similar selective exclusion may occur during the development of GCMs and tuning of parameter values for CMIP experiments [*Mauritsen et al.*, 2012] or in the adaptation of new GCMs from existing models [*Knutti et al.*, 2013]. Given that there is evidence that CMIP GCMs may be biased toward a specific range of climate sensitivities [*Huybers*, 2010], it is still more plausible that there may be a bias against runaway behavior.

As an example, under the extended Representative Concentration Pathway 8.5 (RCP8.5) scenario from the CMIP5 experiments, the CO_2 concentration increases over 250 years to 1962 ppm and is then held constant for 50 years. If we make the approximation that GCMs that would experience a quadratic runaway under a stabilized 1962 ppm CO_2 concentration would also do so under RCP8.5, we can use the quadratic model to estimate what values of λ and a CMIP5 GCMs are necessary to prevent this quadratic runaway. Specifically, for a given a and F, λ will not cause a quadratic runaway if $\lambda < -2\sqrt{Fa}$, and we assume $F_{1962} \approx \log_2(1962\text{ppm}/C_0) \times 3.71$ W/m^2. We can combine this with the CMIP5 ΔT_{4x} range to draw the pink region in Figure 2b, where the curve on the right-hand side is caused by the quadratic runaway cutoff. All CMIP5 GCMs presumably fall inside this region. Since three of the GCMs for which we estimate a fall not far from the curved edge, it would seem that this cutoff is artificial. Further, *Meraner et al.* [2013] find that one of the CMIP5 models (CSIRO-Mk3.6.0) comes close to runaway under RCP8.5.

In our discussion above, we argue that GCMs should not be expected to accurately model warming beyond a certain point, and so GCMs model λ and a much better than higher-order terms. If a GCM never stops warming after being subjected to a CO_2 increase, it may be accurately modeling a pair of λ and a that cause quadratic runaway and therefore may be just as physically accurate as a model that has these same values of λ and a but higher-order terms that happen to cause the model to stabilize.

The second reason we may not often see extreme deviations from equilibrium linearity is that positive a combined with small-magnitude negative λ may be unphysical. As an example, a highly positive low cloud feedback, which would result in a small-magnitude negative λ, could saturate as clouds are lost, causing a negative a. Generally, it is not obvious that a should depend strongly on λ, since the dominant term determining a can be different than the dominant term determining λ (e.g., water vapor feedback versus cloud feedback). More work needs to be done to understand how λ and a covary.

4. Implications for Observational Estimates of Sensitivity and for the Long Tail

Studies often estimate equilibrium climate sensitivity from observations of the recent past [*Gregory et al.*, 2002]. Since the atmosphere equilibrates faster than the oceans, we assume the top-of-atmosphere energy imbalance N is balanced by the ocean heat uptake Q. If we have observations of Q, T, and forcing F for two reference periods, we can estimate λ using

$$\lambda \approx -\frac{\Delta F - \Delta Q}{\Delta T}, \tag{2}$$

as demonstrated by Figure 1c. ΔT_{2x} can then be estimated using the linear model, i.e., $\Delta T_{2x} = -F_{2x}/\lambda$. This estimation of λ assumes that there is no transient nonlinearity and that the climate feedback has not changed due to warming or forcing.

Our knowledge of ΔF, ΔQ, ΔT, and F_{2x} is uncertain, causing uncertainty in our knowledge of λ and ΔT_{2x}. Since many distributions of λ have nonzero probability that λ is arbitrarily close to 0, the linear model implies probability that ΔT_{2x} is arbitrarily large, resulting in a long tail of high climate sensitivities [*Roe and Baker*, 2007].

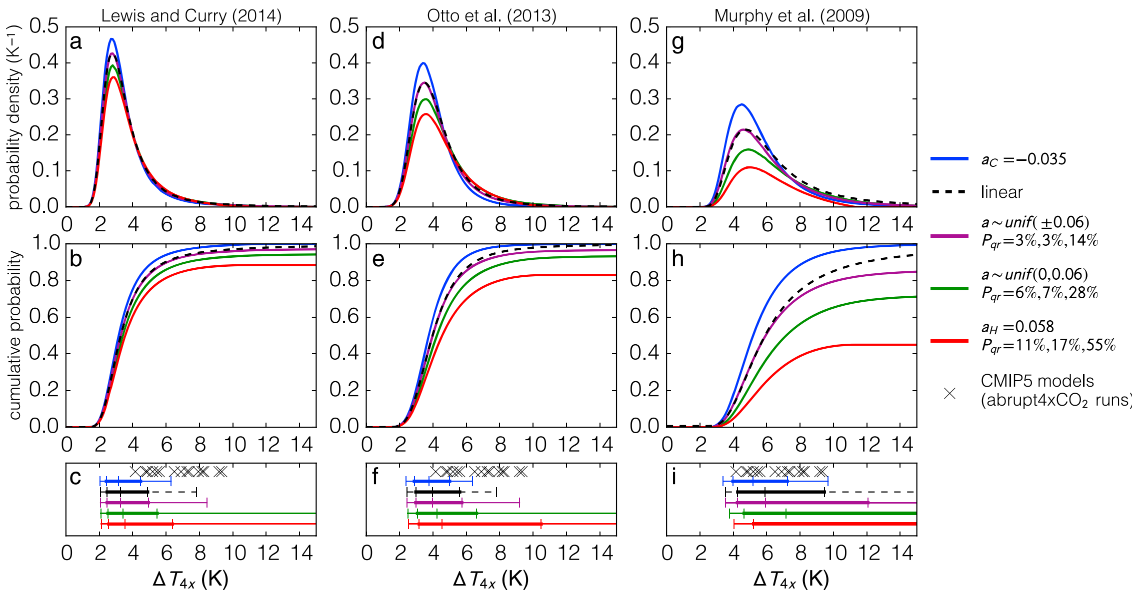

Figure 3. Distributions of ΔT_{4x} recalculated from three observational studies to account for feedback temperature dependence. Linear distributions (dashed black lines) assume a constant feedback, recreating the results from the three studies, while the nonlinear distributions (colored lines) assume a has the minimum (blue lines) or maximum (red lines) value from our analyzed GCMs or assume a follows a uniform distribution covering the range ± 0.06 W/m^2/K^2 (purple lines) or 0 to 0.06 W/m^2/K^2 (green lines). (a, d, and g) Probability density functions and (b, e, and h) cumulative density functions (CDFs) for these distributions. Distributions which allow for positive a (red, purple, and green lines) have a nonzero probability of quadratic runaway (P_{qr}), which causes CDFs to asymptote to values less than 1. These probabilities are listed for each distribution in the legend, with the values for the three studies separated by commas. (c, f, and i) Uncertainty ranges, where ticks demarcate 5th-17th-50th-83rd-95th percentiles, as compared with results from the CMIP5 abrupt4xCO_2 experiment (black crosses).

However, $\lambda \to 0$ does *not* imply $\Delta T_{2x} \to \infty$ but instead that the linear term is disappearing, so that nonlinear terms completely determine the nature of the warming. Accounting for nonlinearity is therefore essential for properly estimating the long tail [e.g., *Zaliapin and Ghil*, 2010].

For a given value or distribution of a, we must alter our analysis in two ways. First, our estimate of λ must account for the feedback change from warming between the reference periods,

$$\lambda \approx -\sqrt{\left(\frac{\Delta F - \Delta Q + a\Delta T^2}{\Delta T}\right)^2 + 4a(F_1 - Q_1)}, \quad (3)$$

where F_1 and Q_1 are values for the first reference period. As Figure 1c demonstrates, this effect is usually negligible, as there has not been enough warming for nonlinearity to act. This makes estimating a from observations of the recent past difficult. Second, we must account for a in estimating long-term warming from λ by using equation (1). We demonstrated in section 3 that this effect can be significant: Figure 1d shows that the scenario that appeared linear in Figure 1c experiences a quadratic runaway under two doublings.

In Figure 3, we use these two alterations to recalculate estimates of the distribution of ΔT_{4x} from three studies: one with a distribution that roughly matches the distribution of CMIP models [*Murphy et al.*, 2009] and two recent studies that find substantially lower climate sensitivities [*Otto et al.*, 2013; *Lewis and Curry*, 2014]. Assuming linearity (dashed black lines) recreates the studies' results (with $\Delta T_{4x} = 2\Delta T_{2x}$). Two of the nonlinear cases (colored lines) assume that a has either the minimum (-0.035 W/m^2/K^2, blue line) or maximum (0.058 W/m^2/K^2, red line) value seen in our GCM survey. The other two nonlinear cases assume a has a uniform distribution of possible values, with a range of either ± 0.06 (purple line) or 0 to 0.06 W/m^2/K^2 (green line). The latter case is included because *Meraner et al.* [2013] argue that a is likely positive (e.g., the majority of CMIP5 GCMs appear to have positive a). Note that in calculating these new distributions of ΔT_{4x}, we are assuming λ and a are independent, which need not be the case, as discussed above.

Distributions that allow for positive values of a (the purple, green, and red lines) have a nonzero probability that Earth will undergo a quadratic runaway for two CO_2 doublings (P_{qr}). Since the quadratic model has an infinite sensitivity in these cases, cumulative density functions of these distributions asymptote to values of $1 - P_{qr}$. As before, knowledge of higher-order terms would allow us to estimate what values of ΔT_{4x}

this probability should be associated with. Since our uncertainty of these terms is large, the proper assignment of this probability is difficult. It is unclear, for example, how much of this probability should fall above or below $\Delta T_{4x} = 12$ K. However, barring large negative higher-order terms, we would expect probabilities such as $P(\Delta T_{4x} > 8$ K) to be accurate.

Adding a temperature-dependent feedback has a small effect on the less sensitive part of the distribution (i.e., the 5th, 17th, and 50th percentiles) but a strong effect on the more sensitive part: $P(\Delta T_{4x} > 8$ K) varies from 2% to 13% for *Lewis and Curry* [2014], from 1% to 20% for *Otto et al.* [2013], and from 11% to 61% for *Murphy et al.* [2009] depending on the assumed value or distribution of a. For all three studies, adding a distribution of a that roughly matches our GCM range (purple line) increases the risk of high warming, despite this distribution having equal likelihood of positive and negative a. If a is likely positive [*Meraner et al.*, 2013], the risk of high warming increases further, as does the risk of quadratic runaway. Accounting for transient nonlinearity would likely further increase the sensitivity of these distributions [*Armour et al.*, 2013].

5. Conclusions

Estimates of the preindustrial climate feedback, λ, and the feedback temperature dependence, a, suggest that equilibrium nonlinearity can strongly affect the warming caused by only a few CO_2 doublings, especially when a is positive. When λ is small-magnitude negative and a is positive, the quadratic model can experience a runaway. In these cases, the amount of warming experienced by Earth depends sensitively on higher-order terms which are difficult to estimate. As a result, if Earth has values of λ and a that cause the quadratic model to run away under a giving forcing, our estimate of the warming response to that forcing may have at best a rough lower bound.

Few GCMs appear to have both small-magnitude negative λ and positive a; therefore, few GCMs exhibit the behavior of extreme equilibrium nonlinearity, e.g., quadratic runaway. Models that experience a quadratic runaway would appear to behave in ways that suggest nonphysical model blowup and may subsequently be selectively excluded from the published literature. Alternatively, combining small-magnitude negative λ and positive a may be unphysical. Future work is need to understand how λ and a covary.

Observational estimates of the risk of high climate sensitivity vary significantly based on the assumed feedback temperature dependence. Published studies that assume linearity likely underestimate the risk of high warming. The long tail contains precisely those quadratic runaway cases described above. For those hoping to constrain the risk of high climate sensitivity, understanding equilibrium nonlinearity is just as essential as understanding λ.

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