Geosci 342 Problem Set 5
Winter Quarter 2009
Due Wed. March 11

March 4, 2009

5 Problem Set:Mostly Shallow Water

Problem 5.1 Consider shallow-water gravity waves in a nonrotating system. Show that for an arbitrary function g, the height field $h = g(x \pm c_o t)$ is an exact solution to the wave equation if $u(x,t)$ is chosen appropriately. Find the corresponding u.

Find a superposition of the two solutions that satisfies $u = 0$ at $t = 0$. Find a superposition that has $h = 0$ at $t = 0$. Describe the evolution of the two cases at later times.

Problem 5.2 Consider shallow-water inertia-gravity waves on the f-plane (i.e. constant f). For a wave with height field $h(x,t) = \cos k(x - ct)$, describe the behavior of $u(x,t)$ and $v(x,t)$ relative to the height field. Present your results in the form of a graph, and describe how the results depend on k.

Why doesn't this wave become geostrophically balanced when the wavelength is long? What is the appropriate Rossby number describing the situation?

Problem 5.3 Quasigeostrophic shallow water flow on the f-plane with a bumpy bottom boundary with height $\eta(x,y)$ is described by the vorticity equation

$$\frac{d}{dt}[\nabla^2\psi + f_o\frac{h-\eta}{H}] = 0 \tag{1}$$

where the geostrophic streamfunction is $\psi = gh/f_o$. h is the free surface height. The relative vorticity is $\nabla^2\psi$ Find the streamfunction, velocity, relative vorticity and free-surface height field caused by steady uniform flow over a bump. Assume the profile of the bump is $\eta(x) = h_o \cos(\frac{1}{2}\pi x/L)$ for $|x/L| < 1$, and $\eta = 0$ otherwise. Discuss how the behavior depends on L.

Hint: First, note that for steady flow that is unsheared far upstream, the quantity in brackets in the vorticity equation has the constant value f_o everywhere. Then, solve the equation for the streamfunction (a second order ODE) separately in the region $|x/L| < 1$ and $|x/L| > 1$. You should be able to do this analytically for both regions. Require that the solution be decaying at large $|x|$. Finally, complete the solution by patching together the solutions in each region, requiring that the streamfunction and the velocity be continuous at $|x/L| = 1$.

Problem 5.4 The quasi-geostrophic shallow-water equations on the β-plane are:

$$\frac{d}{dt}(\nabla^2\psi + f_o\frac{h}{H} + \beta y) = 0 \tag{2}$$

where the geostrophic streamfunction is $\psi = gh/f_o$. Linearize this equation about a state of rest, and find the free Rossby wave dispersion relation. Compare this result with the rigid-lid case. What do you have to do to gravity to make the shallow-water case behave like the rigid-lid case?

Problem 5.5 *Review: Rossby waves forced by wavy wall*

Consider a $\beta - plane$ flow of a layer of fluid with a rigid, flat lid and floor. The fluid is subject to Ekman damping, which is not necessarily small. The only boundary is a wall at the southern boundary, which is slightly wavy, so that the southern boundary is described by $y = \eta(x) = \eta_o \cos kx$, with η_o small. A uniform current U flows through the system in the x-direction. Because the wall is only slightly wavy, the southern boundary condition $v(x) = U\partial_x\eta$ at $y = 0$. The current may be positive or negative.

Find the stationary solution to the linearized problem which decays away from the wall when the Ekman damping is nonzero. Describe what happens when the Ekaman damping is large, and when it is small. Consider both $U > 0$ and $U < 0$. How do the two cases differ?