

Geosci 342 Problem Set 4
Winter Quarter 2009
Due Monday. March 2

March 4, 2009

4 Problem Set: Ekman damping

Problem 4.1 For a frictional boundary layer with friction forces represented as a drag $(-u/\tau, -v/\tau)$ within a layer of depth δ , find the vector (u, v) as a function of τ and δ , and make a vector-plot of how it varies relative to the geostrophic wind (u_g, v_g) as τ is increased from zero to large values.

Problem 4.2 If the boundary layer frictional force is represented in terms of a viscosity, then the equations for the boundary layer flow become

$$\nu \frac{d^2 u}{dz^2} = 2\Omega(v - v_g), \nu \frac{d^2 v}{dz^2} = -2\Omega(u - u_g) \quad (1)$$

Solve these equations for the behavior of the vector (u, v) as a function of z , subject to the boundary condition that $u = u_g$ and $v = v_g$ at large z and $u = v = 0$ at $z = 0$. Show that the velocity vector spirals as one traverses the boundary layer (the "Ekman Spiral"). What is the boundary layer thickness that comes out of this calculation? Compute the divergence of the flow, and the vertical velocity at the upper edge of the boundary layer. Show that it is proportional to the geostrophic vorticity, and compare the coefficient with the one we got from the drag-law formulation.

Hint You can simplify the solution of this problem by introducing the complex variable $U = u + iv$ and rewriting the pair of second order equations as a single second order equation involving U .

Problem 4.3 Including the effects of Ekman damping, the stationary Rossby wave equation for flow over a mountain is

$$U \partial_x \nabla^2 \psi + \beta \partial_x \psi + 2\Omega U \partial_x h/H = -E \nabla^2 \psi \quad (2)$$

Solve this equation for a mountain of shape $h = h_o \cos kx \sin \ell y$. The flow is in a channel on the β -plane, and has $v = 0$ at $y = 0$ and $y = \pi/L$. How does the Ekman damping affect the phase of the streamfunction (pressure) relative to the mountain? What does the damping do to the resonance that used to occur in the undamped case when $k^2 + \ell^2 = k_s^2$?

Problem 4.4 Consider a circular ocean basin on a β -plane. The radius of the ocean basin is r , and it is driven by a wind with profile $U_o \cos \frac{1}{2}\pi y/r$, with $y = 0$ taken to be the center of the circle.

Assuming Sverdrup balance, find the flow in this ocean basin assuming that there is no flow through the boundary on the right-hand side of the basin. Where do you expect boundary currents to occur? Note that in this problem, you do not assume that $u = 0$ on the right-handd boundary. Instead, you must assume u takes on the value that makes the normal component of the velocity vanish at the boundary.