Geosci 342 Problem Set 3
Winter Quarter 2009
Due Wed. Feb. 18

February 9, 2009

3 Problem Set:Mostly Rossby Waves

Problem 3.1 Consider a fluid on a rotating plane ($\beta = 0$) which is initially at rest. There is a ridge $h(x)$ at the bottom boundary, with $h = h_o \exp(-x^2/L^2)$. At time $t = 0$ a uniform flow U in the x-direction is suddenly turned on and the fluid begins to ascend the upwind slope and descend the downwind slope.

(a) After a short time ΔT has passed, what does the vorticity look like? Explain your sketch in terms of vortex stretching.

(b) After a long time has passed, what does the vorticity look like over the ridge? Sketch the direction the streamlines bend over the ridge. You need not carry out the integral to determine the streamfunction, however; just getting the general shape right is OK. For the purposes of this problem you may assume that the flow is an undisturbed uniform current far upstream. Using reasoning based on vortex stretching, show that a region of cyclonic vorticity appears on the lee side of the mountain, and moves downwind as time progresses. Sketch the streamlines at several different times, taking into account this strip of cyclonic vorticity.

Problem 3.2 For midlatitude motions on Earth, the effective value of β is about $10^{-11} m^{-1} s^{-1}$, in dimensional terms. Consider a wave in a current of $30 m/s$, which is confined in a channel with width such that the effective north-south wavenumber $\ell = 10^{-6} m/s$. The zonal (east-west) wavelength is 10000 km. What is the zonal wavenumber? Using the Rossby wave dispersion relation, determine which direction a ridge or trough of this wave moves, and determine how long it takes for a ridge or trough to move 5000 km.

Problem 3.3 Consider *easterly* flow (i.e. $U < 0$) on the β plane over a sinusoidal mountain $h(x) = h_o \sin(kx)$. What is the streamfunction field? Is there any phase shift between the ridges in the streamfunction and the ridges in the mountain? Where is the vorticity cyclonic? Where is it anticyclonic? Explain this pattern in terms of vortex stretching/compression, and the generation of vorticity by north-south motions acting on the planet's vorticity gradient (the β-effect).

Now, imagine that the flow has started up from rest, so that at $t = 0$ the flow is a uniform undisturbed current U. In order to satisfy this initial condition, you must superpose the steady solution with a free Rossby wave which cancels out the steady solution at the initial instant of time. The free (i.e. unforced) wave solution satisfies the Rossby wave dispersion relation, and so it evolves in time. Describe how the superposition of the free and forced wave evolves in time. Does the system ever reach a steady state? Speculate on why not, and what additional physical processes might allow the system to reach a steady state after a long time has passed.