Errata and Updates for Principles of Planetary Climate

Last updated $**CURRENT^{**}$

This document contains errata and updates to the first edition of *Principles* of *Planetary Climate* by Raymond T. Pierrehumbert. Items marked *Erratum* correct misprints and other errors in the text. Items not so marked are updates or extensions of the treatment in the original text, which will eventually be incorporated in the Second Edition. Items are tagged with the date (Euro format) they were added to the file. Updated and supplementary Workbook problems will be collected in a separate document. ii

Preface

The Big Questions

p. 23, line after Eq. (1.2), ; Added 11/26/2012

It should be pointed out that the exponent 3.5 in the mass-luminosity scaling law applies over most of the Main Sequence, but goes down to about 2.3 for low-mass stars, below about $0.4M_{\odot}$.

Eq. (1.3), p. 23 (Erratum); Added 11/26/2012

The exponent in the equation should be .71

p. 23, line after Eq. (1.2), ; Added 11/26/2012

p. 24 (Erratum) Added 2/17/2013

The phrase "or ice giants like Neptune or Saturn" in the third complete paragraph should be "or ice giants like Uranus or Neptune"

1.7.2, p. 29 (Erratum)

There was a typographical error in the number quoted for the deuterium to hydrogen ratio of the outer Sun, but beyond that the reasoning in the subsequent part of the sentence was muddled. The sentence "This differs ... ; ... same as Jupiter." should be replaced by:

All the deuterium in the Universe was created during the Big Bang, and the amount has been decreasing ever since, as stars destroy it in the course of thermonuclear fusion. It is estimated that the protosolar deuterium to hydrogen ratio is 1 in 50000. Jupiter's atmosphere is about 10% enriched compared to this value, but Earth's ocean water is much more enriched, which may provide a clue as to the nature of the planetesimals that delivered Earth's water. It could also be that Earth at some stage preferentially lost the lighter isotope through escape to space.

Thermodynamics in a nutshell

Table 2.2, p. 92 (Erratum)

The critical point pressure for NH_3 was stated as bars instead of being converted to Pa as it should have been. The critical point pressure entry for NH_3 should be 112.8e5. This has also been corrected in the corresponding entry of phys.py

Addition 11/8/2012: In the caption to the table, the temperature stated for the maximum density of pure liquid water should have been +4C instead of -4C.

p 98, first paragraph and Eq. (2.23) (Erratum); Added 11/26/2012

The discussion of dry static energy is incorrect. The dry static energy is not in general an exact differential for the First Law, and is only conserved for adiabatic motions. (The problem comes about because the transformation from p to z using the hydrostatic relation only works if ρ can be written as a function of p alone, as is the case for adiabatic motion, i.e $\delta Q = 0$.)

The paragraph beginning "The hydrostatic relation ...", and the subsequent paragraph beginning "Note that the dry static energy ..." should be replaced with the following modification, which now includes a discussion of enthalpy conservation during convection:

The hydrostatic relation also allows us to derive a conserved energy-like quantity for adiabatic motion, which can serve as a useful alternate to the potential temperature. For adiabatic motion of an air parcel, the constancy of potential temperature implies that ρ can be written as a function of p, given the potential temperature of the parcel. In that case, we can define a height

z(p) via the relation $gdz/dp = -1/\rho$. This transformation can actually be done whether or not the hydrostatic approximation applies, but when the system is hydrostatic and the motion occurs in an environment of uniform potential temperature, z will be the actual altitude of the air parcel. This situation would apply, for example, when an air parcel ascends or descends in a region where convection has set the temperature profile to the dry adiabat. Using the transformation, for adiabatic motion the First Law can be re-written

$$\delta Q = 0 = c_p dT - \rho^{-1} dp = c_p dT - \rho^{-1} \frac{dp}{dz} dz = d(c_p T + gz)$$
(2.23)

assuming c_p to be constant. The quantity $c_pT + gz$ is known as the *dry static* energy.

Since we had to assume adiabatic motion in order to derive the conservation law for dry static energy, dry static energy does not provide a convenient basis for determining what happens when heat is added to or taken away from a column of the atmosphere. Consider a column of atmosphere which exchanges no mass with its neighbors, and suppose an amount of heat $\delta Q(p)$ is added at each level pin the column. How does the temperature change? Once more, the hydrostatic approximation comes into play. Since adding heat to any portion of the column does not change the mass of that parcel of atmosphere, the hydrostatic relation implies that the pressure at which any given parcel of atmosphere is located does not change, though the *altitude* of that parcel changes as the column expands or contracts in response to the heating. Hence, dp = 0 following any individual parcel in the course of this process, and so,

$$\delta Q = d(c_p T) \tag{2.23a}$$

assuming c_p to be constant. The quantity c_pT is known as the *dry enthalpy*. Dry enthalpy provides a more convenient basis for atmospheric energy budgets than entropy, since changes in dry enthalpy in an isolated column of the atmosphere are equal to the net energy added to or removed from the column by heat sources such as solar radiation. For example, if there are no horizontal transports and if there is no net flux of energy between the atmosphere and the underlying planetary surface, then the rate of change of the net dry enthalpy in a (dry) atmospheric column is the difference between the rate at which solar energy flows into the top of the atmosphere; one needs to know nothing about how the heat is deposited within the atmosphere in order to determine how the net dry enthalpy changes. This is not the case for entropy. Since convection merely re-arranges the heat content of a column without adding any net energy, the net dry enthalpy of a column remains fixed in the course of (dry) convection.

Note that the dry enthalpy as defined above is actually the enthalpy *per* unit mass of atmosphere. Thus, the total enthalpy in a column of atmosphere, per unit surface area, is $\int c_p T \rho dz$, which by the hydrostatic relation is equal to $\int c_p T dp/g$ if the pressure integral is taken in the direction of increasing pressure. More generally, the mass-weighted vertical integral of any quantity in a hydrostatic atmosphere is equal to the integral with respect to pressure, divided by the acceleration of gravity.

p 108 (Erratum); Added 6/12/2013

The two sentences beginning "When the moist adiabatic slope is lower than ..." and " Conversely, ..." should be replaced by:

When the absolute value of the moist adiabatic slope is less than R/c_p , then lifting a parcel adiabatically creates enough cooling that the parcel becomes supersaturated, and condensation occurs. Conversely, if the absolute value of the moist adiabatic slope is greater than R/c_p , condensation occurs on descent instead.

Section 2.7.3, p 111-112 (Erratum); Added 11/26/2012

The discussion of moist static energy needs to be corrected along the lines discussed for dry static energy. Most of the discussion can be fixed by simply restricting the applicability to adiabatic motions $\delta Q = 0$, but it is more useful to expand the discussion to include moist enthalpy. The section heading should be changed to "Moist static energy and moist enthalpy," and the body of the section should be replaced by:

When the condensable substance is dilute, it is easy to define a moist enthalpy and moist static energy which generalize the corresponding dry quantities defined in Eq. (2.23a) and Eq. (2.23). This is accomplished by multiplying Eq. (2.29) by T to obtain the heat budget, dropping the terms proportional to r_c (which are small in the dilute limit), and making use of $R_a T/p_a = 1/\rho_a \approx 1/\rho$. For adiabatic motion the hydrostatic relation is used to re-write the pressure work term in precisely the same way as was done for dry static energy. Then, if we further assume that the temperature range of interest is small enough that L may be regarded as constant, we obtain

$$\delta Q = dh - \frac{1}{\rho} dp$$

$$0 = d(h + gz), \text{ for } \delta Q = 0$$
(2.34)

where $h \equiv c_{pa}T + Lr_c$ is the moist enthalpy per unit mass. Note that these relations do not assume that the condensable mixing ratio r_c is saturated. For adiabatic flow in the dilute case, the dilute moist static energy will be conserved

following an air parcel whether or not condensation occurs. In the general case, with $\delta Q \neq 0$, we can still constrain the change in enthalpy in a column of atmosphere in hydrostatic balance, which exchanges no mass with its surroundings. Because little mass is lost during condensation in the dilute limit, the hydrostatic relation implies that $dp \approx 0$ following each fluid parcel in the column, as in the dry case. In consequence, if energy is added to or taken away from the column, then the change in the net moist enthalpy of the column is equal to the net amount of energy added or taken away.

When the condensate is non-dilute, things are a bit more complicated. In this case significant amounts of mass can be lost from the atmosphere in the course of condensation, and in essence the precipitation of condensate can take away significant amounts of energy with it. In order to deal with the heat storage in condensate, one must make use of the specific heat of the condensed phase, which we will refer to as $c_{pc\ell}$ (regardless of whether the condensate is liquid or solid); the behavior of this specific heat is inextricably linked to the changes in latent heat with temperature through the thermodynamic relation

$$\frac{dL}{dT} = c_{pc} - c_{pc\ell} \tag{2.35}$$

which is valid when the condensate density is much greater than the vapor density. This relation is essentially a consequence of energy conservation. In the following we'll neglect the temperature dependence of the specific heats in order to simplify the algebra, though Eq. (2.35) remains valid even if the specific heats are temperature-dependent. Eq. (2.35) will be used to allow for the temperature-dependence of latent heat in the thermodynamic conservation laws.

First let's analyze the energy budget per unit mass of the non-condensable substance. We'll write the pressure work term in Eq. (2.28) in the alternate form $(m_a/\rho_a)dp$. On dividing the equation by m_a and using Eq. (2.35) and the hydrostatic relation to rewrite the pressure work in the adiabatic case, we obtain

$$(1+r_c)\delta Q = d\mathfrak{h} - \frac{1}{\rho_a}dp - c_{pc\ell}Tdr_c, \ \mathfrak{h} \equiv (c_{pa} + r_c c_{pc\ell})T + L(T)r_c$$

$$0 = d[\mathfrak{h} + (1+r_c)gz] - (c_{pc\ell}T + gz)dr_c, \ \text{for } \delta Q = 0$$

$$(2.36)$$

where \mathfrak{h} is the moist enthalpy per unit mass of non-condensible substance. The quantity in square brackets in the second line is the moist static energy per unit mass of non-condensable substance; it will be denoted by the symbol \mathfrak{M} . In each equation, the term involving dr_c , represents the sink of moist enthalpy or moist static energy due to the heat and potential energy carried away by the condensate.

The form of moist enthalpy given above can be inconvenient to use, because the $(1+r_c)$ weighting on the left hand side makes it hard to do the energy budget of a column of air knowing only the net input of energy into the column. The expression also becomes inconvenient when the atmosphere becomes dominated by the condensable substance, leading to very large values of r_c . We can formulate the moist enthalpy per unit *total* mass of the gas by dividing Eq. (2.36) by $(1 + r_c)$. After carrying out a few basic manipulations, we find

$$\delta Q = d \left[\frac{\mathfrak{h}}{1 + r_c} \right] - \left(c_{pc\ell} T - \frac{\mathfrak{h}}{1 + r_c} \right) d \ln(1 + r_c) - \frac{1}{\rho} dp.$$
(2.37)

The term $\mathfrak{h}/(1+r_c)$ is the desired moist enthalpy per unit of total gaseous mass, and the second term is the corresponding sink due to precipitation. This equation can be used to constrain the temperature changes in an isolated column of atmosphere in response to heating, cooling or heat redistribution by convection, much as was done in the dry case or the dilute moist case. To do this in the non-dilute moist case, however one must confront two important complications. First, one must explicitly take into account the loss of moist enthalpy due to precipitation. Second, it is no longer valid to assume that dp = 0 following a fluid parcel, even for an isolated atmospheric column, since mass loss due to condensation above the fluid parcel reduces its pressure, and the corresponding pressure work also needs to be taken into account in computing the change in moist enthalpy of the column.

Exercise 2.15 Re-write the expression $\mathfrak{h}/(1+r_c)$ in terms of the mass specific concentration q. What is the form of this expression when $q \approx 1$? What happens to the precipitation sink term in this limit?

An alternate approach to dealing with moist enthalpy in the non-dilute case is to write an energy budget per unit total mass (condensate included) for an air parcel that retains its condensate. To allow for precipitation, one then deals explicitly with the energy loss occurring when some of the retained condensate is removed from the air parcel. This approach is especially useful in situations when the mass of retained condensate can be appreciable. One must still account for the pressure work done when the column expands to compensate for the reduction in hydrostatic pressure that occurs when condensate is removed. The retained-condensate approach can be carried out using the moist entropy expressions derived in Emanuel (1994) (see Further Reading). Further modifications to the expression for moist enthalpy are required if both ice and liquid phases are present in the atmosphere, in order to account for the latent heat of the solid/liquid phase transition.

The moist enthalpy budget imposes an important constraint on the evolution of temperature in an isolated atmospheric column, though application of the constraint is straightforward only in the dilute limit. As we shall see in Chapter 9, moist static energy and moist enthalpy also form a convenient basis for diagnosing horizontal atmospheric energy flows, and for formulating idealized energy balance models incorporating geographic variations of temperature and other atmospheric properties.

Prob. 2.56, p. 131 Added 2/17/2013

The phrase "Plot and discuss results for the mixing ratio" should be "Plot and discuss results for the molar concentration". There is nothing wrong with discussing the mixing ratio instead, but MoistAdiabat.py computes the concentration, not mixing ratio, and there's no particular reason to go to the additional step of converting this to a mixing ratio.

Elementary models of radiation balance

10 CHAPTER 3. ELEMENTARY MODELS OF RADIATION BALANCE

Radiative transfer in temperature-stratified atmospheres

Fig. 4.11, p. 223 (Erratum)

In the water molecule shown in the upper left hand panel of the figure , the H and O symbols were switched in the molecular structure drawing. The corrected figure is below:



Section 4.4.3, p. 226 6/12/2013

Following the sentence in the last paragraph "In addition, the lines of a molecule in motion... molecule moving away" insert the sentence

Similarly, as seen by an observer at rest, the frequency of emitted light will be shifted higher or lower accordingly as the molecule is moving towards or away from the observer. Also, the following sentences should be appended at the end of the paragraph, following the statement of the Doppler-broadened CO_2 line width:

However since the Doppler-broadened width is proportional to the line center frequency, Doppler broadening tends to become more important for high frequency (short wavelength) lines. This can have a significant effect on the heating of the upper portions of atmospheres by absorption of shortwave incoming stellar radiation.

Eq. (4.62) p. 228 and surrounding text (Erratum) See Addendum 6/12/2013

There was an error in the description of the algebraic prefactor multiplying the exponential in the equation for temperature scaling of line strength. The short form of the correction is to replace the text "where n is the line-width exponent defined above" by "where n is -1 for linear molecules like CO_2 and -1.5 for nonlinear molecules like H_2O " At the end of the paragraph, one should also insert the caveat: "Eq. (4.62) is reasonably accurate for temperatures under

500K, and when $h\nu_c/kT$ and $h\nu_c/kT_0$ are not small." However, I've learned a lot more about line strength since writing this paragraph, and I now prefer the following more complete discussion of line strength, which should replace the paragraph surrounding Eq. (4.62):

The line intensities are independent of pressure, but they do increase with temperature. Since the emission or absorption depends on the probability with which a transition from one state to another occurs, the line intensity depends on the probability that the higher and lower states in the transition are occupied; as always in equilibrium thermodynamics, this probability is determined by the equipartion principle, stating that each state gets an equal share of energy. Implementation of the principle using standard methods of statistical thermodynamics yields the following expression for line strength:

$$S(T) = S(T_0) \frac{Q(T_0)}{Q(T)} \left[\frac{1 - \exp\left(-\frac{h\nu_c}{kT}\right)}{1 - \exp\left(-\frac{h\nu_c}{kT_0}\right)} \right] \exp\left(-\frac{h\nu_\ell}{k} \left(\frac{1}{T} - \frac{1}{T_0}\right)\right)$$

where Q(T) is a function called the *total internal partition function* which depends on the distribution of energy states of the molecule. $h\nu_{\ell}$ is the energy of the lower energy state in the transition that gives rise to the line. This energy is tabulated in standard spectroscopic databases, and is usually stated as the frequency ν_{ℓ} . Determination of the lower state energy is a formidable task,

since it means that one must assign an observed spectral line to a specific transition. When such an assignment cannot be made, one cannot determine the temperature dependence of the strength of the corresponding line.

When the temperature is not too high, $Q(T) \sim T$ for linear molecules like CO_2 and $Q(T) \sim T^{1.5}$ for nonlinear molecules like H_2O) or CH_4 . At large temperatures, Q(T) becomes somewhat larger than the power law scaling would predict; this happens for T > 1000K for most molecules, though the power law breaks down around 400K for CO_2 . For high accuracy in hot atmospheres, one should use the actual Q(T) tabulated as part of spectroscope databases like HITRAN (discussed at the end of this section), but since the temperature scaling is dominated by the exponential term given by the final factor in the term in brackets in Eq. (4.62) is close to unity so long as $h\nu_c/kT$ and $h\nu_c/kT_0$ are sufficiently large, which is typically the case unless one is extrapolating to frequencies much lower than the peak of the Planck function.

The version of PyTran.py used to make the spectral survey plots in the text and to make the exponential sum tables did have the incorrect form of the algebraic prefactor, but because the temperature scaling was used only to scale from the 296K standard HITRAN temperature to 260K, and because the temperature scaling is dominated by the exponential term, the differences in the corrected plots are hardly visible. For computations done using the (slightly) inaccurate exponential sum tables, the small change due to the correction in the algebraic prefactor is completely negligible in comparison with the inaccuracies introduced by the approximations used to handle temperature scaling in the band models (e.g., use of a exponential temperature factor for all bands). Still, for research purposes, it would be best to recompute the exponential sum tables using the updated version of PyTran.py. The script PyTran.py has been updated to incorporate the correction in temperature scaling, currently using the approximate power law form but in future releases with implementation of the actual Q(T) for selected molecules.

Addendum 6/12/2013: A bug was introduced in the course of implementing the corrected temperature scaling. See the comments in the ComputeAbsorption function of Pytran for details. The courseware web site has been updated with the corrected version. I have checked the Pytran output against the independently written Fortran code kspectrum, and for Lorentz line shapes it gives the same results. Note that the exponential sum tables in the WorkbookDatasets collection are still the original versions used in the text. They have not yet been replaced with recomputed tables. The existing tables can still be used in teaching, but for research purposes tables should be recomputed using the newest version of Pytran (and probably also with the most current version of the HITRAN database).

Eq. (4.85) , p. 241 (Erratum)

The expression for B_j on the right hand side of the first line of the equation should be multiplied by π to take into account angle averaging.

Eq. (4.91), p 260 (Erratum)

The expression 10-8 multiplying ν^3 should be $10^{-8}.$

Prob. 4.19, p. 308 (Erratum) Added 2/17/2013

The expression $4\sigma T_g^3$ in the last line should be $4\sigma T_{rad}^3$

Scattering

Fig. 5.2 and assoc. discussion, p 321 Added 2/17/2013

The discussion of this figure could be misread as implying that the number of scatterers in the shaded parallelogram is larger than in the unshaded rectangle by a factor of $1/\cos\theta$; of course, the area (and number of scatterers) of the two regions are identical. What the discussion is meant to imply is that the contribution of these scatterers to the flux $I(\theta)$ traveling in direction θ is amplified by $1/\cos\theta$. The reason is that I gives the flux through a unit area normal to the direction of travel, and this area is smaller than the area of the top of the parallelogram by a factor $\cos\theta$. I am working on an improved version of the figure and discussion that makes this budget easier to understand; the path of least resistance is to draw in the projected area normal to the beam in direction θ , but this makes an already cluttered figure even more cluttered. Once I have something I'm satisfied with, I'll update this item with the revised figure and discussion.

Eq (5.19), p. 325 (Erratum) Added 2/17/2013

The coefficient multiplying the direct beam term on the right hand side should be $-\omega_0$ instead of $+\omega_0$. This does not affect any subsequent calculations since the correct form of the equation was used in computing the coefficients for the two-stream approximations.

Eq (5.21) and following text to end of paragraph, p. 326 (Erratum+update) Added 2/17/2013

A step was left out of the derivation of Eq. (5.21). The two terms proportional to \tilde{g} in this equation are exact, but the remaining term on the rhs requires an assumption about the angular distribution of I in order to be written in terms of $I_+ - I_-$. Using the hemi-isotropic assumption the factor in parentheses multiplying $I_+ - I_-$ should be $(2 - \omega_0 \frac{3}{2}\tilde{g})$ instead of $(1 - \omega_0 \frac{3}{2}\tilde{g})$. In addition, the coefficient multiplying the direct beam term should be $-\omega_0$, not $+\omega_0$. This does not affect the two-stream equations used in the rest of the chapter, since the two-stream coefficients were calculated from the correct form of Eq. (5.21), not the one printed in the text. I'll also use the opportunity of this revision to streamline the discussion and move the remarks about the hemi-isotropic approximation out of this paragraph and into the two-stream section where it belongs. The equation and the discussion in the remainder of the paragraph should be replaced by:

$$\frac{d}{d\tau^*}(I_+ + I_-) = -\left(\int_{\Omega^+} Id\Omega - \int_{\Omega^-} Id\Omega\right) + \omega_0 \frac{3}{2}\tilde{g}(I_+ - I_-) - \omega_0 L_{\circledast} \frac{3}{2}\tilde{g}\cos\zeta \exp\left(-\left(\tau_{\infty}^* - \tau^*\right)/\cos\zeta\right).$$
(5.21)

The asymmetry terms have the same form as in Eq. (5.19) apart from the factor $\frac{3}{2}$. In this case, the left hand side is already written in terms of $I_+ + I_-$, without the need for any approximation. However, to obtain a two-stream closure, the two terms on the first line of the right hand side need to be expressed in terms of I_+ and I_- by making some assumption about the angular distribution.

Paragraph following Eq. (5.29), p 339 (Erratum+update) Added 2/17/2013

The expression for $\gamma_+ - \gamma_-$ in the tenth line of this paragraph should be $-2\gamma\omega_0\hat{g}\cos\zeta$, i.e the $\cos\zeta$ factor was left out. The expressions in Eq. (5.30) are correct and the misprint does not affect any of the calculations in this chapter since those were done using the correct expressions in Eq. (5.30). In addition for the hemi-isotropic case, the two-stream asymmetry factor is related to the standard asymmetry factor by $\hat{g} = \frac{3}{4}\tilde{g}$, not $\frac{3}{2}$ as originally stated in the text. This relation was never used in any of the subsequent calculations. For improved clarity, and for consistency with the changes made in (5.21) and the

subsequent discussion, the portion of the paragraph after Eq. (5.29) beginning "In consequence ..." through to the end of the paragraph should ve replaced by:

When the asymmetry function $G(\cos\theta)$ can be represented as being proportional to $\cos \theta$, it then follows that $\gamma_1 + \gamma_2 = 2\gamma \cdot (1 - \hat{g}\omega_0)$, where γ is related to the proportionality coefficients used in approximating $I_+ + I_-$ and $I_+ - I_$ and \hat{g} is a coefficient characterizing the asymmetry of the scattering. Further, comparing the direct beam terms between Eq. (5.29) and Eq. (5.15) implies $\gamma_+ - \gamma_- = -2\gamma\omega_0\hat{g}\cos\zeta$. If $H = \cos\theta$, then $G = \tilde{g}\cos\theta$ exactly and so \hat{g} is in fact the asymmetry factor \tilde{g} defined by Eq. (5.17). For other forms of H the assumed proportionality of G to $\cos\theta$ is only approximately valid, but this assumption introduces errors that are no worse than other errors that are inevitable in reducing the full scattering equation down to two streams. The precise form of the resulting asymmetry factor depends on the assumptions made in approximating G, but the values tend to be reasonably close to \tilde{g} . For example, closing the two-stream approximation by assuming the flux to be isotropic (i.e. constant I) within the upward and downward hemispheres implies $I_{+} = \int_{\Omega_{+}} I \cos \theta d\Omega = \frac{1}{2} \int_{\Omega_{+}} I d\Omega$ and similarly for I_{-} , whence $\gamma = \gamma' = 1$ if we use the form of H given by Eq. (5.20) to do the asymmetric projection. If we further assume a phase function truncated to its first three Fourier components, $G = \frac{3}{2}\tilde{g}\cos\theta$, whence Eq. (5.21) implies that $\hat{g} = \frac{3}{4}\tilde{g}$.

Eq. (5.42) and nearby text, p. 345 (Erratum)

Two lines above Eq. (5.42) the expression given in the text for $I_+ + I_-$ should be $I_+ + I_- = 2\pi B - (1 + 2\gamma \tau^*)I_{+,\infty}$. Eq. (5.42) should be

$$I_{+,\infty} = \pi B / \left(1 + \gamma \tau_{\infty}^* \right).$$

The expression following "infrared by a factor" in the line following the equation should be $1/(1 + \gamma \tau_{\infty}^*)$

Eq. 5.52, p. 349 (Erratum) Added 6/12/2013

A factor L_{\circledast} should be inserted before the exponential in the last term of the equation.

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The surface energy balance

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Variation of temperature with season and latitude

p. 445 (Erratum) Added 11/26/2012

In the final paragraph, the portion beginning with "This is where the notion ..." up to "...is simply $\mu \equiv c_p p_s/g$." should be replaced with:

This is where the notion of moist or dry enthalpy, introduced in Chapter 2, comes into its own. For simplicity, let's consider a non-condensing atmosphere, for which case the enthalpy (per unit mass) is c_pT , and assume T(p) is given by the dry adiabat throughout the atmosphere. The net column enthalpy is then $c_pT_{sa} \cdot (1 + R/c_p)^{-1}p_s/g$, where T_{sa} is the near-surface air temperature. Hence, the energy required to change the surface air temperature by 1 K while keeping the potential temperature well mixed in the vertical is simply $\mu \equiv c_p \cdot (1 + R/c_p)^{-1}p_s/g$.

p. 446 (Erratum) Added 11/26/2012

In the paragraph beginning "It is convenient ..." the values of H_{eq} should be 1.9m for Earth, .026m for the current Mars atmosphere, 167m for Venus, 8.5m for Early Mars, and 21m for Titan.

p. 453, Table 7.1 (Erratum) Added 6/12/2013

The units for conductivity in the table header should be $(W/(m \cdot K))$

Prob. 7.15, p. 486 (Erratum) Added 11/26/2012

In the first paragraph, change "static energy" to "enthalpy" and replace the second paragraph by:

In the first limit, we consider the condensable substance to be dilute, as is the case for water vapor on the present Earth. In this case we can use the dilute moist enthalpy formula given in Section 2.7.3. Suppose that the stratosphere has negligible heat storage, and that T(p) can be assumed to be given by the moist adiabat for the purpose of computing thermal inertia. Suppose that the condensable substance is saturated at all heights. Find an expression for the energy storage as a function of surface temperature. The thermal inertia coefficient μ is the derivative of this with respect to surface temperature. Evaluate this at 260 K, 280 K, 300 K, and 320 K for saturated water vapor on Earth. Evaluate it for saturated methane in an N_2 atmosphere on Titan, for a suitable range of temperatures. Express your answer in terms of an equivalent mixed-layer depth of liquid water. How do the values compare with what you would get in the same situations using dry enthalpy? Note that when the atmosphere is unsaturated, the latent heat term cannot be written as a function of temperature, and the determination of the thermal inertia coefficient is correspondingly more complicated.

p. 458 (Erratum) Added 6/12/2013

The phrase "the ratio of solar constant at apastron to that at periastron is 1+4e" should be replaced by "the ratio of stellar constant at periastron to that at apastron is 1+4e"

Prob. 7.16, p. 487 (Erratum) Added 11/26/2012

 $c_p p_s/g$ in the next-to-last paragraph of the problem should be $(c_p/(1+R/c_p))p_s/g$

Evolution of the atmosphere

Eq. (8.9), p. 502 (Erratum)

In the equation for the water dissociation equilibrium, the ion OH^- should be enclosed in square brackets, i.e. $[OH^-]$, to indicate that it represents a concentration.

Eq. (8.16), p. p 516, second line (Erratum)

Denominator should be $[HCO_3^-]$, i.e. use square brackets to denote that the quantity is a concentration of the bicarbonate ion.

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A peek at dynamics

Section 9.2, p 603 Added 11/26/2012

In deriving the atmospheric energy budget, I had been reaching for a way to avoid introducing the momentum equations, but I over-reached and my trick didn't work since the moist static energy is not an exact differential for the First Law. The correction results in a small modification of the atmospheric energy storage form (but not the energy *flux*), and since the transient energy storage was not actually used in any calculation and appears only tangentially in the discussion, none of the results are affected. I will try to come up with a more elegant treatment of this derivation in the Second Edition, but meanwhile, the derivation can be repaired by replacing the paragraph following Eq. (9.6), which begins "To obtain the zonal mean energy equation ...", and the rest of the text on the page up through Eq (9.10) with the following:

To obtain the zonal mean energy equation, we apply Eq. (9.6) to the moist static energy per unit mass, \mathfrak{M} , defined in Section 2.7.3. We will restrict attention to the case in which the concentration of the condensable component is small, so that the simplified form of the moist static energy applies and we need not worry about the loss of moist static energy through precipitation. The source term of the column integral of \mathfrak{M} is obtained by rewriting the First Law following a fluid parcel. Specifically, suppose that the net vertical flux of energy by all means other than large scale transport is F(p) at pressure level p. Then, the energy source per unit mass per unit time is $-g\partial_p F$, with the convention that downward fluxes are positive. The First Law then becomes

$$\frac{d}{dt}(c_{pa}T + Lr_c) - \frac{1}{\rho}\omega = -g\partial_p F$$

In order to rewrite the pressure work term as the divergence of a flux, we need

to bring in one result from the horizontal momentum equation, which we have not so far needed to consider. Using the momentum equation, it can be shown that the neglect of the kinetic energy term in the energy budget is equivalent to the statement that

$$(\frac{1}{a\cos\phi}u\partial_{\lambda} + \frac{1}{a}v\partial_{\phi})gz \approx 0$$

This equation states that the work done by horizontal pressure gradient forces is negligible. With this result in hand, a little manipulation making use of the hydrostatic relation yields

$$-\frac{1}{\rho}\omega = \omega\partial_p gz \approx (\frac{1}{a\cos\phi}u\partial_\lambda + \frac{1}{a}v\partial_\phi)gz + \omega\partial_p gz = \frac{d}{dt}gz - \partial_t gz$$

whence

$$\frac{d}{dt}\mathfrak{M} = -g\partial_p F + \partial_t gz$$

where $\mathfrak{M} \equiv c_{pa}T + gz + Lr_c$ is the dilute form of the moist static energy.

At the top of the atmosphere, let $F_{\circledast,top}(\phi,t)$ be the net zonal mean solar flux into the atmosphere (downward positive), and $OLR(\phi,t)$ be the zonal mean outgoing thermal infrared (upward positive, as usual). At the surface, let $F_{\circledast,s}(\phi,t)$ be the zonal mean solar flux exiting the bottom of the atmosphere; the difference with $F_{\circledast,top}(\phi,t)$ gives the atmospheric solar absorption. The heat flux out of the bottom of the atmosphere consists of turbulent as well as infrared radiative terms; call this net flux simply F_s , with the convention that a positive value indicates a transfer of heat from the atmosphere to the underlying surface. With these definitions the vertically integrated source term, which gives the rate of change of moist static energy in a column, becomes

$$\left\langle \int_{0}^{p_{s}} \mathfrak{S}_{B} \frac{dp}{g} \right\rangle = F_{\circledast,top} - OLR - F_{\circledast,s} - F_{s} + \partial_{t} \left\langle \int_{0}^{p_{s}} gz \frac{dp}{g} \right\rangle \tag{9.7}$$

The form of the final term assumes that z = 0 at the bottom boundary, i.e. a flat surface; otherwise the exchange of ∂_t and the vertical integral would require the addition of a boundary term involving the mountain height. Substituting the source term into Eq. (9.6), and using the moist static energy in place of B, we obtain the energy balance equation

$$\partial_t E_{atm} + \frac{1}{\cos\phi} \partial_\phi \Phi_{atm} \cos\phi = F_{\circledast,top} - OLR - F_{\circledast,s} - F_s \tag{9.8}$$

where we have defined the atmospheric energy flux as

$$\Phi_{atm} \equiv \frac{1}{a} \left\langle \int_0^{p_s} v \mathfrak{M} \frac{dp}{g} \right\rangle \tag{9.9}$$

and the mean heat storage in the atmospheric column as

$$E_{atm} \equiv \left\langle \int_0^{p_s} h \frac{dp}{g} \right\rangle. \tag{9.10}$$

where h is the dilute moist enthalpy per unit mass, $c_{pa}T + Lr_c$. Note that the final term in Eq. (9.7) has canceled out the contribution of gz in the time derivative of moist static energy, with the result that the atmospheric heat storage is the vertically integrated *moist enthalpy* rather than the vertically integrated *moist static energy*. Nonetheless, it is the convergence of the flux of moist static energy that causes the enthalpy in a column to change.

Problem 9.16, p 633 Added 11/26/2012

Replace the phrase "mean moist static energy $\underline{E_{atm}}$ defined in Eq. (9.10)" with "column-integrated dilute moist static energy $\overline{\mathfrak{M}}$ ", and replace subsequent references to E_{atm} with $\overline{\mathfrak{M}}$.