QM6

ATOMIC PHYSICS

Recommended Reading

There isn't a single textbook which will meet all your needs for the A paper Atomic Physics. You will probably need to refer at different times to several of the books on the following list.

- Cohen-Tannoudji, Diu and Laloe, Quantum Mechanics.
- Robinett, Quantum Mechanics
- Pauling and Wilson, Introduction to Quantum Mechanics. The treatment of hydrogen and helium is recommended for its clarity and detail — but some of the stuff about electron spin is misleading.
- Bransden and Joachain, Physics of Atoms and Molecules. Probably the best of the more recent texts.
- Haken and Wolf, Atomic and Quantum Physics. There's a lot of useful stuff in here, but it follows a rather different order.
- Cagnac and Pebay-Peyroula, Modern Atomic Physics (in two volumes). Similar remarks to Haken and Wolf.
- Woodgate, Elementary Atomic Structure. This excellent book was *not* written as an introductory text, and is not suitable on its own for the second year course.
- Series, The Spectrum of Atomic Hydrogen (originally an OUP monograph 1957, but re-issued with new material in 1988 by World Scientific). This is a more advanced text, but contains all the anwers for hydrogen.

The conclusion to this is that you have to find several books, and hunt out the material you want at the level you understand. Please take this warning very seriously, or you will have problems with the atomic physics.

(1) Quantum Mechanics of Hydrogen

The aim of this week's work is to become familiar with the *wavefunctions* of the hydrogen atom, their quantum numbers, shapes and degeneracies. You can work entirely in the x-representation, but don't get bogged down in the intricacies of the series solutions. Or you can follow the Binney algebraic approach — but make sure you end up knowing what all the wavefunctions do, not just the maximum angular momentum ones.

Write an account of the quantum mechanics of hydrogen-like atoms, that is, atoms with one electron. Include in your account:

- The approximations involved in the choice of Hamiltonian;
- The solutions of the Schrodinger equation, energy levels and quantum numbers, spectroscopic notation, atomic units;
- Scaling properties of the solutions (*i.e.* scaling of length, energy and normalization) with (*e.g.*) reduced mass, nuclear charge;
- A discussion of the physical properties of the solutions. There are many things you could choose to include here, such as
  - · Detailed sketches of some of the wavefunctions, *e.g.* ψ along the $\pm Z$ axis for $1s$, $2s$, $2p$ $m_l = 0$ $3d$ $m_l = 0$ states (computer-generated plots, three-dimensional visualizations, movies etc welcomed!)
  - · The form of the wavefunction near the nucleus for s, p and d states; amplitude of s wavefunctions at the origin; physical reason for the suppression of the wavefunction near the origin in some states ('centrifugal barrier');
  - · Properties of the angular part of the wavefunction, angular momentum; diagrams showing the length and direction of the angular momentum vector in p and d states;
  - · Properties of the radial part of the wavefunction, probability density and radial probability density; expectation values such as $\langle r\rangle$, $\langle 1/r\rangle$ (there are formulae for these in the books, but you could easily work them out for some simpler states);
  - · etc., etc.

  Make your own choice from these and other topics.

(2) 1997 8. The radial Schrödinger equation for an electron in the Coulomb field of a point charge Ze can be written

$$\left[-\frac{\hbar^2}{2m_{\mathrm{e}}}\frac{d^2}{dr^2} - \frac{Ze^2}{4\pi\epsilon_0 r} + \frac{\ell(\ell+1)\hbar^2}{2m_{\mathrm{e}}r^2}\right] P = EP, \qquad \text{where } P = rR(r).$$

What do the various terms signify? How is $R(r)$ related to the complete wavefunction $\psi(r,\theta,\varphi)$?

Verify that $R(r) = \exp(-Ar)$ and $R(r) = r\exp(-Ar/2)$, where A is a constant which you should identify, yield solutions of the radial Schrödinger equation, and find the corresponding energies and angular momenta.

Taking the charge Ze to be distributed uniformly over the surface of a small sphere of radius a ($a \ll 1/A$) find in first approximation how the energy of the ground state depends upon the value of a.