1. INTRODUCTION

Our work is devoted to a new mechanism of the amplification and generation of coherent radiation, based on the use of systems with strong electromagnetic (em) field–matter coupling and without population inversion.

What is strong field–matter coupling? First of all, strong coupling between field oscillators and matter oscillators means that the corresponding coupling constant exceeds the rate of incoherent relaxation. It is known that the coupling constant between an em field and matter is small for a single atom. Therefore, to increase such coupling, we must use either a cavity with a very high quality factor or many atoms—in the case of a bad cavity or free space—because the coupling strength is proportional to the square root of the atomic density for a passive medium, this quantity coincides with the density of absorbing atoms. Therefore, the strong coupling, in our case, is a collective effect [1], and the parameter of the strong coupling coincides with the so-called cooperative frequency of the matter

\[ \omega_c = \frac{\sqrt{\frac{2\pi d^2}{\hbar}} \omega_0 n}{\lambda} > 1/T_2, \]  

where \( \omega_0 \) is the spectral transition frequency, \( d \) is the spectral transition dipole moment, and \( T_2 \) is the state coherence time for a two-level system. This means, in particular, that the exchange of energy between the field and matter occurs faster than the processes of irreversible relaxation. It is supposed also that the coherence of both the em field and matter is important.

In general, the cooperative frequency is included in many formulas of optics and laser physics. First of all, we have to mention the classical electronic theory of dispersion (the Lorenz electron theory of optics) and the concept of an optical Langmuir plasma frequency. In the current literature, the physical meaning of the optical plasma frequency is treated, as a rule, depending on the particular conditions of the problem: (1) field–matter coupling constant [2], (2) cooperative frequency, (3) frequency of energy exchange between the field oscillators and matter oscillators [3], (4) space–time scale of coherence in Dicke superradiance, (5) so-called supernutation frequency or frequency of the Bloch-pendulum small vibrations near the equilibrium position [4], and (6) self-induced Rabi oscillation frequency [5].

For the strong em field–matter coupling, the weakness of the driving field is important; i.e.,

\[ E^2/4\pi < n\hbar \nu. \]  

(This estimation is valid for the simplest case of a single-mode cavity.) Then, the reaction of matter, namely, its reemission, plays an essential role during the dynamical interaction of the field and matter, and the driving field does not destroy the collective coupling between matter oscillators (dipole–dipole interaction through a field mode).

Finally, both photon and matter components of the system play an equal role during their interaction. This means, from the quantum point of view, that we deal with the polariton. The presence of the strong field–matter coupling has serious consequences for the dynamics of the behavior of the coupled field–matter oscillators:

(1) There is a possibility for spontaneous coherence in the field–matter system (possibility for self-organization) [6]. In particular, the spontaneous periodical exchange of energy between the field and matter has
been shown (at least on the matter-coherence time scale $T_2$). It is natural that there is some spatial scale of such a coherence. Again, from the quantum point of view, we have the appearance of cavity polaritons [7].

(2) There is self-splitting of a resonator bar mode (the appearance of the normal modes for the system of the em field–matter coupled oscillators; cf. Fig. 1a). Here, we have a very close relationship with some results of the cavity QED [8].

(3) There is self-splitting of the dispersion curve (cf. Fig. 1b) of a spectral transition of the matter (more correctly, of the compound field–matter system). In the quasiparticle language, one can speak of the appearance of upper and lower polariton dispersion branches.

(4) We have an opportunity for coherent radiation parametric amplification and generation in the considered field–matter system without population inversion, which arises as a result of coupling modulation (in practice, it is reduced to modulation of the ground- or metastable-level population of an absorbing medium). In other words, we have a possibility for the parametric amplification and generation of cooperative field–matter oscillations—a polariton amplifier and generator [9, 10].

(5) There is a possibility for nonclassical light generation (squeezing, entanglement) [11].

(6) Finally, in a system with strong field–matter coupling, there exists a possibility for the realization of a polariton Bose–Einstein condensate at room temperature [12].

2. SPECTRUM CONDENSATION PHENOMENA UNDER INTRACAVITY PUMPING OF AN OPTICALLY DENSE RESONANT MEDIUM: AN EXAMPLE OF THE STRONG FIELD–MATTER COUPLING PARAMETRIC EXCITATION

For the first time, we have faced the question about the influence of strong coupling on the dynamics of a radiating system in a study of so-called spectrum condensation under intracavity pumping of an optically dense medium. The experimental setup includes a resonator with a broadband amplifying cell (in our case, a pulsed dye cell) and a narrow-band resonantly absorbing cell also placed inside a resonator. A detailed description of the experiment and its results are given in [9].

In that case, as an absorbing medium, we have used neon metastable atoms arising in the positive column of a glow discharge and in the afterglow of a pulsed discharge (the density was $10^{12}–10^{13}$ cm$^{-3}$). The atoms have strong absorption lines (the corresponding oscillator strengths were approximately several tenths) in the red–yellow area of the spectrum, which is convenient for registration. Under certain conditions (the realization of threshold values for the pump intensity and absorbing particle density), the generation of this system is concentrated near the strongest absorption lines of the medium (see Fig. 2). It is especially necessary to emphasize that such a phenomenon of spectrum condensation is significant only near the strongest lines, and it is practically absent near the weak lines (this is

![Fig. 1.](image-url)
clear in Fig. 2a). In many cases, the condensed spectrum has a doublet structure. In Fig. 2b, one can see the dependence of the doublet splitting on the absorbing atom density. In some cases, for certain conditions and, in particular, for increased pumping power of the active medium, the condensation spectrum becomes complicated: instead of two basic components of the condensation spectrum placed on both sides of an absorption line, additional components appeared that were approximately equally spaced from each other in frequency (cf. Fig. 3, where spectrum condensation near the spectral transition of a neon line at 594.5 nm is shown). It is interesting to note for this case that the pump spectrum (a radiation spectrum of the dye cell) is mainly outside the spectrum doublet of the condensation.

The phenomenon of spectrum condensation under intracavity laser pumping of an optically dense medium is a universal phenomenon. It was observed by us as well as by other authors in sodium, potassium, helium, and in rare-earth elements (see references in [8]). Lasers of many types have been used also. Recently, rather interesting data were obtained in molecular systems [13].

Fig. 2. Spectrum condensation in neon: (a) generation at different spectral lines; (b) self-splitting of the generation spectrum at different time moments in the afterglow of a pulsed discharge, which corresponds to different absorbing metastable atom densities in the intracavity absorbing cell, $\lambda = 594.5$ nm.
metastable state (about 10⁻¹³ cm⁻³). The high density of Ne in the plasma in a Ne glow discharge and plasma of the pulsed (without a resonator) was carried out for the cases of nontarget media without population inversion in free space coherent propagation and amplification of polychromatic light 

Some treatments were based on effects such as a linear and nonlinear lens, competition of cavity modes, phenomena caused by the occurrence of a prism due to a density gradient, etc. We suppose that there is, first of all, a fundamental reason for the phenomena observed. The main reason is the parametric excitation of the strong coupling by its modulation (modulation of a coupling parameter) [9, 10].

3. AMPLIFICATION AND GENERATION OF POLYCHROMATIC LIGHT IN AN OPTICALLY DENSE EXTENDED MEDIUM WITHOUT POPULATION INVERSION

An experimental and theoretical investigation of the coherent propagation and amplification of polychromatic broadband laser pulses in optically dense resonant media without population inversion in free space (without a resonator) was carried out for the cases of plasma in a Ne glow discharge and plasma of the pulsed discharge afterglow. The high density of Ne in the metastable state (about 10¹²⁻¹³ cm⁻³), which was chosen to be the ground state of the two-level system under consideration, leads to the collective behavior of an atomic system in a resonant electromagnetic field. The broadband probe field attenuation and the sideband amplification (without population inversion) of the probe under the action of a coherent pump were observed. The amplification dependence on the pump intensity was obtained. A model of the parametrically enhanced collective interactions was developed, and a comparison with experimental data was made. Details of the theory and experiments on free-space parametric interactions in the strong-coupling regime can be found in [14, 15].

A principle scheme of the experimental setup is presented in Fig. 4. The multimode dye laser tuned to the one of resonant transitions under investigation was taken as the source of the coherent radiation. The laser beam was split into two parts (the probe field and the pump) by a beam splitter formed by a parallel-side plate and lenses. These two beams were intersected inside the discharge tube at a small angle that can be varied in the range from 0.5° to 1.5°. The probe beam intensity amounted to 2% of the intensity of the pump. The maximal value of the pump flux density was equal to 15 kW/cm². In some cases, we used two independent probe and pump beams from two synchronized dye lasers. The output probe-field spectrum was recorded using an optical multichannel analyzer and a Fabry–Perot interferometer piezoelectrically scanned with a spectral resolution of 200 MHz.

In the case of the neon glow-discharge plasma, the experiments were carried out at three different Ne spectral lines: 585.2 nm (1s₂ → 2p₁), 588.2 nm (1s₂ → 2p₂), and 594.5 nm (1s₂ → 2p₀). The 1s₂ level is the lowest metastable state of the neon atom, whereas the 1s₂ level is the quasi-metastable one due to radiation trapping. The parameters of the discharge had the following values: neon pressure p = 1 Torr, discharge current i = 20–50 mA, discharge tube radius R = 0.4 cm, and length L = 15 cm (the length of the beam intersection region was 4 cm; the radii of the laser beams were 0.05 cm). The densities of atoms at the lower state of the transitions under consideration are n = 6 × 10¹¹ cm⁻³ and n = 1.5 × 10¹¹ cm⁻³ for the 1s₂ and 1s₂ states, respectively. The duration of one dye laser pulse was 3.5 ns, which is substantially shorter than the homogeneous relaxation times. The spectral bandwidth of multimode generation was equal to 40 GHz, which also exceeds the inhomogeneous broadening of the resonant line. The dye laser longitudinal modes (the intermode distance was 0.37 GHz) were not phase-locked, resulting in quasi-stochastic laser pulse generation. We measured the input and output spectra of the probe field both in the absence and in the presence of the pump field at different values of the field intensities and detunings between the generation spectrum maximum and the transition frequency while varying the discharge conditions.

Figure 5 shows the process of the coherent amplification of the polychromatic probe field under the action of the pump beam field. We attribute the appearance of two amplification maxima to the parametric enhancement of the cooperative radiation, which, under certain conditions, can lead to the excitation of the cooperative parametric resonance.

An analysis of the interaction between short polychromatic pulses and an optically dense extended reso-
nant medium was carried out on the basis of the theory of coherent transient phenomena of the light–matter interaction, taking into account the collective behavior of an atomic system. The theory of transient processes of interaction between an electromagnetic field and resonant media is based on the semiclassical model. We restrict our study to the two-level system approximation, assuming that interaction between the field and neighboring atomic transitions is negligibly small.

A one-dimensional problem of the interaction between an ensemble of two-level atoms and two plane linearly polarized waves intersecting at an angle \( \varphi \) under the continuous medium approximation was considered. One can obtain the following system of Maxwell–Bloch equations describing the theoretical model under analysis:

\[
\begin{align*}
\frac{c \cos \varphi}{\partial z} \frac{\partial \Omega_{R1}}{\partial z} + \frac{\partial \Omega_{R1}}{\partial t} &= \omega_p^2 p_1, \\
\frac{c}{\partial z} \frac{\partial \Omega_{R2}}{\partial z} + \frac{\partial \Omega_{R2}}{\partial t} &= \omega_p^2 p_2, \\
\frac{\partial p_1}{\partial t} &= -\Omega_{R1} D - \frac{p_1}{T_2}, \\
\frac{\partial p_2}{\partial t} &= -\Omega_{R2} D - \frac{p_2}{T_2}, \\
\frac{\partial D}{\partial t} &= \frac{1}{2} \Omega_{R1}^* p_1 + \frac{1}{2} \Omega_{R2}^* p_2 + \text{c.c.} - \frac{D - D_0}{T_1},
\end{align*}
\]

where the following designations have been made: \( c \) is the speed of light in vacuum; \( \Omega_{R1,2}(t, z) = 2d \varepsilon_{1,2} / \hbar \) are the complex Rabi frequencies of correspondent fields (\( d \) is the electric dipole moment of the atomic transition); \( p_{1,2}(t, z) \) are the complex polarizations of atoms at the point \( z \) at the time \( t \) induced by the correspondent fields; \( D(t, z) \) is the population difference of the atoms;

![Fig. 4. Experimental setup: (1) Cu–vapor pump laser, (2) dye laser, (3) spectrograph, (4) laser beam splitter, (5) aperture, (6) scanned Fabry–Perot interferometer, (7) photoelectron multiplier, (8) lenses, (9) optical fiber, (10) driven screens, (11) discharge tube, (12) amplifiers, (13) control computer, (14) discharge tube power supply.](image1)

![Fig. 5. Transmission of the probe laser field in the absence of the pump beam (1) and its amplification in the presence of the pump (2); Ne, \( \lambda = 640.2 \text{ nm} \).](image2)
\(D_0\) is the value of \(D\) in the absence of external field (the value \(D = 1\) corresponds to an atom in the ground state); and \(\omega_c\) is cooperative frequency (1), which plays the role of the coupling coefficient between field and matter. The relaxation times \(T_1\) and \(T_2\) were included phenomenologically. In the derivation of system (3), the slowly varying envelope approximation was used; the Doppler broadening of spectral line was not taken into account.

Equations (3) describe the propagation of two intersecting plane waves that are coupled by the population difference of a two-level medium.

We numerically studied propagation of polychromatic broadband quasi-stochastic laser radiation in a dense medium without population inversion. The parameters of the numerical simulations were similar to the experimental conditions. The input electric fields have the following form:

\[
\Omega_R(t, 0) = C(t) \sum_{k = -N}^{N} \Omega_{0k} e^{-i(\Delta_{2k} + \omega_0 t + \alpha_k)},
\]

\[
\Omega_{0k} = \Omega_{00} e^{-i(4\ln2)\omega_0 / \gamma},
\]

\[
C(t) = \frac{2}{\gamma} e^{-i(t - t_0)/\gamma} (\pi/2 + \arctan((t - t_0)/\gamma)),
\]

where \(\omega_0 = k\Delta_0 \alpha (k = 0, \pm 1, \ldots, \pm N)\) are the modes of the input spectrum, \(\Delta_0\) is the intermode distance, \(\Delta_{2k}\) is the detuning between the central frequency of the field and that of the atomic resonance, and \(\alpha_k\) are the mode phases. The phases are random numbers, which leads to the quasi-stochastic temporal dependence of the electric field. The duration of this signal does not depend on the width of the spectrum \(\gamma\) but rather is determined by the duration of the envelope \(C(t)\). The following parameters of the input signal were used: \(\Delta_0 / 2\pi = 0.37\) GHz, \(\gamma / 2\pi = 20.0\) GHz, duration of the envelope \(C(t) = 3.5\) ns. The amplitude of the probe field was chosen to be seven times smaller than that of the pump, and a short (about 10-ps) time delay between the fields was taken into account.

First of all, we would like to mention the importance of collective phenomena accompanying the propagation of a single coherent pulse in an optically dense medium without population inversion. Under the approximation of a linear medium \(D(t, z) = 1\), the solution for the broadband electromagnetic field entering the initially unperturbed medium at the point \(z = 0\) is given by

\[
\Omega_R(t, z) = \Omega_R(\tau, 0) \int_0^{\tau} \Omega_R(\tau - \tau', 0) e^{-i/\gamma z/c} \omega_c \sqrt{\gamma(\omega_c^2 - \chi_0^2)} J_1(2\omega_c \sqrt{\gamma(\omega_c^2 - \chi_0^2)}) d\tau',
\]

where \(\tau = t - z/c\) and \(J_1(x)\) is a Bessel function. The integral kernel presents a Green’s function of the problem and, thus, the response of the atomic ensemble to a \(\delta(t)\) pulse. This oscillating response has a superradiant character and appears in a system of a strongly coupled field and matter, where the high density of resonant atoms provides the atomic system with the ability to coherently absorb and reradiate the entire energy of the external electromagnetic field. Such oscillations display the process of photon interchanges between a field and a two-level atomic system in an extended resonant medium and the formation of a 0π pulse. The regularity of the process is caused by the appearance of a correlation between dipoles on a macroscopic scale due to dipole interaction through the reemission field. Contrary to the effects in a strong field, where the macroscopic dipole correlation appears due to synchronization by the external field, the field of the matter reaction plays a key role in weak-field phenomena, giving rise to the cooperative or collective behavior of a macroscopic system. The cooperative interactions are transient phenomena and can be observed when the frequency of the photon interchanges between matter and the field exceeds the relaxation rates of a medium.

Expression (5) shows that optical ringing is not a harmonic process; nevertheless, it is possible to estimate the characteristic duration of the collective radiation pulses in the initial stage of the signal: in a cross section with the longitudinal coordinate \(z\), for the case of a broadband field, the frequency of collective interaction takes the form

\[
\Omega_z = \omega_z^2 c / \gamma;
\]

i.e., it is proportional to the number of interacting atoms (in this case, only propagation in the forward direction is considered); the analogous frequency in the case of spatially homogeneous interaction is equal to the cooperative frequency of the medium \(\omega_c\). On the other hand, for a given time moment \(t\), the spatial distribution of the field \(\Omega_R(t, z)\) also has a quasi-periodic structure and the cooperative grating exists in an extended medium. It is important to notice that the generation of the cooperative ringing and the cooperative grating are mutual processes.

The collective ringing phenomena can be described in terms of the beating between the wavepackets of photons-in-matter (polaritons) of different group velocities. On the other hand, such a quantum-electrodynamics phenomenon as the vacuum Rabi splitting under the creation of a photon–atom bound state at the level of the “single atom–single mode” cavity interaction, described by the Jaynes–Cummings model, has a direct macroscopic analogy in the collective phenomena in a dense medium, since, in both cases, the following two conditions are fulfilled: (i) the number of photons is smaller than the number of interacting atoms and (ii) the field–matter coupling coefficient exceeds the rates of incoherent relaxation.
Considering the interaction of two copropagating waves with a resonant medium on the basis of Maxwell–Bloch equations (3), we assume that wave 1, propagating at the small angle of \( \phi \) to the \( z \) axis, is a weak probe field and that wave 2 is a strong pump propagating in the \( z \) direction.

By differentiating first equation in Eq. (3) with respect to time and taking into account third equation in Eq. (3), one can obtain an equation describing the temporal and spatial evolution of the probe field amplitude \( \Omega_{R1}(t, z) \):

\[
\frac{\partial^2 \Omega_{R1}}{\partial t^2} + \frac{1}{T_2} \frac{\partial \Omega_{R1}}{\partial t} + \omega_c^2 D(t, z) \Omega_{R1} = 0
\]

In the case of a spatially homogeneous system, the analogous equation takes the form \( \dot{\Omega}_{R1}(t) + 1/T_2 \dot{\Omega}_{R1}(t) + \omega_c^2 D(t) \Omega_{R1}(t) = 0 \) and represents the parametric oscillations of a pendulum. Under the periodic modulation of the resonance frequency, this second-order ordinary differential equation is the Hill equation, and, in the case of a harmonic modulation, this is a Mathieu equation. In systems that can be described by the Hill equation, the effect of parametric resonance can be observed and exponential growth of the solution appears. Second-order differential equation (7) represents a generalization of the Hill equation for the case of a spatially inhomogeneous system.

The population difference \( D(t, z) \) is a parameter that modulates the coupling coefficient between the field and the matter. If the pump amplitude \( \Omega_{R2} \) greatly exceeds the amplitude of the probe field \( \Omega_{R1} \), the dynamics of the population difference is entirely determined by the pump field, meaning that one can consider the effects of the parametric amplification of the probe field by the pump energy, and it is the quantity \( \omega_c^2 D(t, z) \) that couples these fields.

We center our discussion on the case in which the role of the pump field is reduced to a small modulation of the coefficient that couples the probe field and mat-
eter. In contrast to the case of strong-field phenomena, which can be considered in the framework of the single-driven-atom model, under these conditions the pump does not destroy the collective dipole–dipole interaction of atoms via the photons of the probe field but rather parametrically enhances it. This type of population modulation is related to the oscillations of the collective Bloch vector near the equilibrium point, which corresponds to the ground state of an atom \((D = 1)\). Such oscillations are possible under an interaction with a field of small area (either a weak field or a relatively strong field with a dip in the spectrum at the frequency of an atomic transition). Under this condition, the function \(D\) can vary quasi-periodically and does not change sign, which means that the medium can stay uninverted. In Eq. (7) for the probe field, this corresponds to the quasi-periodical dependence of the frequency of oscillations on small modulations of the amplitude. This is the type of probe field amplification that is analogous to the parametric resonance described by the Hill equation. The peculiarity of such an amplification mechanism consists in the fact that no population inversion in a two-level atomic system is required for its appearance and maintenance.

Figure 6 presents the probe amplification due to the parametrically enhanced collective interaction. In this case, the amplification mechanism can be presented as follows. During the propagation of the probe and pump fields, the process of the correlated oscillations of the electromagnetic fields and population difference arises due to the cooperative effects of the photon interactions between the field and matter. The frequency of such oscillations is of the order of frequency (6) and increases during the propagation. In the system of oscillating field \(\Omega_{21}(t, z)\) and population difference \(D(t, z)\), a parametric resonance appears, which leads to the probe amplitude increase at the frequencies of the oscillations. Thus, the peculiarity of the amplification process under the cooperative parametric resonance consists in the presence of two significant maxima in the output signal spectrum at frequencies that correspond to the frequencies of collective interaction between an ensemble of two-level atoms and a weak resonant field (see Fig. 6). In the figures, the output probe signals in the absence and in the presence of the pump are compared. It is shown that the degree of temporal regularization of the electromagnetic field increases strongly in the case of the parametric excitation of the collective effects. The figures show that, during the amplification process, the atomic system stays near its equilibrium state (no inversion is necessary). In accordance with the analogy to the splitting phenomena of the medium dispersion and of the quasi-levels of the bound matter–field system mentioned above, this amplification can be treated as coherent Ramanlike scattering that is due to collective self-splitting of the medium energy levels.

We attribute the experimental spectra of the probe sideband amplification to the model of the parametric enhancement of the collective interactions presented. The intracavity phenomenon of spectrum condensation is also directly related to this mechanism.

4. CONCLUSIONS

Sources of coherent light–matter coupling and without population inversion are proposed. Such sources can be made for any spectral range, including UV. They will have an extremely low pump threshold and a very high amplification coefficient. Their realization is especially interesting in microcavities containing different nanostructures [16].

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