Quantum Club

Assignment 6

TENSOR PRODUCT SPACES.

- Study Sec. 2.1 of the textbook and solve the exercises therein.

Problem 1. Two photons are initially prepared in one of the four Bell state. The polarization of both photons is rotated by angle θ. For example, for the state $|\Psi^-\rangle$ this results in the state (see Ex. 2.9)

$$\left|\Psi^-(\theta)\right\rangle = \frac{1}{\sqrt{2}}\left(|\theta\rangle \otimes \left|\frac{\pi}{2}+\theta\right\rangle - \left|\frac{\pi}{2}+\theta\right\rangle \otimes |\theta\rangle\right). \tag{1}$$

Write similar expressions for the other three Bell states and then express them in the canonical basis. Discuss how the Bell states transform into one another when θ is varied.

Problem 2. End-of-chapter problem 2.1 for $|\Phi^+\rangle$.

Problem 3. End-of-chapter problem 2.2.

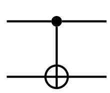
Problem 4. End-of-chapter problem 2.4.
Hint: Your Hamiltonian matrix will be of the form

$$\begin{pmatrix} \square & & & \\ & \square & \square & \\ & \square & \square & \\ & & & \square \end{pmatrix},$$

where $\square$ are nonzero elements. Think of ways to simplify the diagonalization of this matrix arising from its "block-diagonal" shape.

QUANTUM COMPUTING 2.

- Study Sec. 2.5 of the textbook and solve the exercises therein, except 2.64.
- Launch the Composer on the IBM quantum computing web site. As this assignment deals with 2-qubit circuits in the assignment, you will need to remove all extra qubits in the Composer quantum circuit window except $q[0]$ and $q[1]$. The qubit $q[0]$ is the least significant qubit, meaning that it corresponds to the rightmost digit in the basis states.
- An additional gate required in this assignment is the C-not gate.

  The control qubit (top in the diagram above) is marked with a thick dot and the target qubit is marked by the cross. You can use your mouse to drag the dot with respect to the cross to choose the desired control qubit.

Problem 5. Design and test the quantum circuits to prepare each of the four Bell states.

Problem 6. Design and test a quantum circuit that would enable a measurement in the Bell basis — that is, implement the following transformation on a two-qubit Hilbert space:

$$\begin{aligned} \left|\Psi^-\right\rangle &\to |00\rangle\,; \\ \left|\Psi^+\right\rangle &\to |10\rangle\,; \\ \left|\Phi^-\right\rangle &\to |01\rangle\,; \\ \left|\Phi^+\right\rangle &\to |11\rangle\,. \end{aligned} \tag{2}$$

Problem 7. Design and test a quantum circuit that would enable a conditional phase gate as defined in the book.

LOCAL MEASUREMENTS.

- Study Sec. 2.2 of the textbook and solve the exercises therein.

Problem 8. End-of-chapter problem 2.6.

Problem 9. End-of-chapter problem 2.7.

Problem 10. End-of-chapter problem 2.8. The Greenberger–Horne–Zeilinger state is

$$|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle).$$