Quantum Club

Assignment 5

UNITARY OPERATORS AND OPERATOR FUNCTIONS.

- Study Secs. A.10 and A. 11 of the textbook and solve the exercises therein.

Problem 1. Find the matrices of $\exp(i\phi\hat{A})$ and $\sqrt{\hat{A}}$, where the matrix of $\hat{A}$ is given in

a) Problem 1.15;

b) Exercise A.65(a).

For (a), write the initial operator $\hat{A}$ and the answers in the Dirac notation, assuming that the matrix of $\hat{A}$ is given in the canonical polarization basis.

Which of the resulting operators are Hermitian, and which are unitary? Give an argument why you expect this to be the case. For the unitary operators, check explicitly that they are inverse to their adjoints.

EVOLUTION AND SCHRÖDINGER EQUATION.

- Study Sec. 1.10 of the textbook and solve the exercises therein. "Method III" of finding the state evolution [Eq. (1.32)] is not required.

Problem 2. An atom is described in some basis $\{|v_1\rangle, |v_2\rangle\}$ by the Hamiltonian

$$\hat{H} = \hbar\omega \begin{pmatrix} 1 & 2i \\ -2i & 4 \end{pmatrix}.$$

a) Find the energy eigenstates and eigenvalues.

b) The state of the atom is $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|v_1\rangle + i\,|v_2\rangle)$. For the energy observable being measured in this state, find the probabilities of detecting each energy eigenvalue, as well as the mean and variance of this measurement.

c) The atom is initially in the state $|\psi(t=0)\rangle = |\psi_0\rangle$. Find its state $|\psi(t)\rangle$ at an arbitrary time t using both Methods I and II introduced in Sec. 1.10.

d) Check explicitly that the state $|\psi(t)\rangle$ that you found obeys the Schrödinger equation.

e) How much time will elapse until the atom is once again in the state $|\psi_0\rangle$ (up to a phase factor)?

Problem 3. End-of-chapter problem 1.16 (a,b).

Problem 4. End-of-chapter problem 1.17.

Problem 5. End-of-chapter problem 1.18.