Quantum Club

Assignment 3

MATRIX PRODUCT.

- Look up the rule for matrix multiplication (e.g. `https://www.youtube.com/watch?v=2spTnAiQg4M`).

Problem 1. Find the matrix $\hat{A}^n$, if the matrix $\hat{A}$ is

a) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$; b) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$; c) $\begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_N \end{pmatrix}$; d) $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$.

ADJOINT SPACES.

- Study Sec. A.5 of the textbook and solve the exercises therein.

LINEAR MAPS, OPERATORS, MATRICES.

- Study Sec. A.6 of the textbook and solve the exercises therein.

Problem 2. Two operators have the following matrices in the canonical basis: $\hat{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$; $\hat{B} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$. Write down the vectors $\hat{A}\,|H\rangle\,, \hat{A}\,|V\rangle\,, \hat{B}\,|H\rangle\,, \hat{B}\,|V\rangle$ in the Dirac notation. Use these results to compute $\hat{A}\hat{B}\,|H\rangle\,, \hat{A}\hat{B}\,|V\rangle$ and hence write down the matrix of the operator $\hat{A}\hat{B}$ using Eq. (A.19) from the book. Compare the result with the matrix of $\hat{A}\hat{B}$ calculated using the matrix product rule.

Problem 3. Check the resolution of the identity explicitly for the circular basis: express $|R\rangle\langle R|$ and $|L\rangle\langle L|$ in the canonical basis and check that the sum of these operators equals identity.

Problem 4. Use the resolution of the identity method (Sec. A.6) to express the Pauli operators (1.7)

a) $\hat{\sigma}_x$ in the diagonal basis;

b) $\hat{\sigma}_y$ in the circular basis.

Problem 5. Problem 1.5 from the textbook (end of Chapter 1), except part (g).

OPERATORS IN QUANTUM PHYSICS.

- Study Sec. 1.7 of the textbook and solve the exercises therein.

Problem 6. Problem 1.6 from the textbook.

Problem 7. Problem 1.7 from the textbook.

Problem 8. Problem 1.8 from the textbook.

QUANTUM COMPUTING 1.

- In any web browser, open the website `http://quantum-computing.ibm.com`.
- Create an IBMid account and log in.
- When logged in, click on the button 'Launch Composer' on the left-hand side. If this does not work, you can also go to the webpage `http://quantum-computing.ibm.com/composer` instead.
- Multiple windows should appear. Please close the window labeled 'OpenQASM 2.0' or 'Qiskit'.
- Change the window labeled 'Q-sphere' to a 'Statevector' window by clicking on 'Q-sphere' and selecting 'Statevector'.

If all is correct your screen should look like Fig. 1.

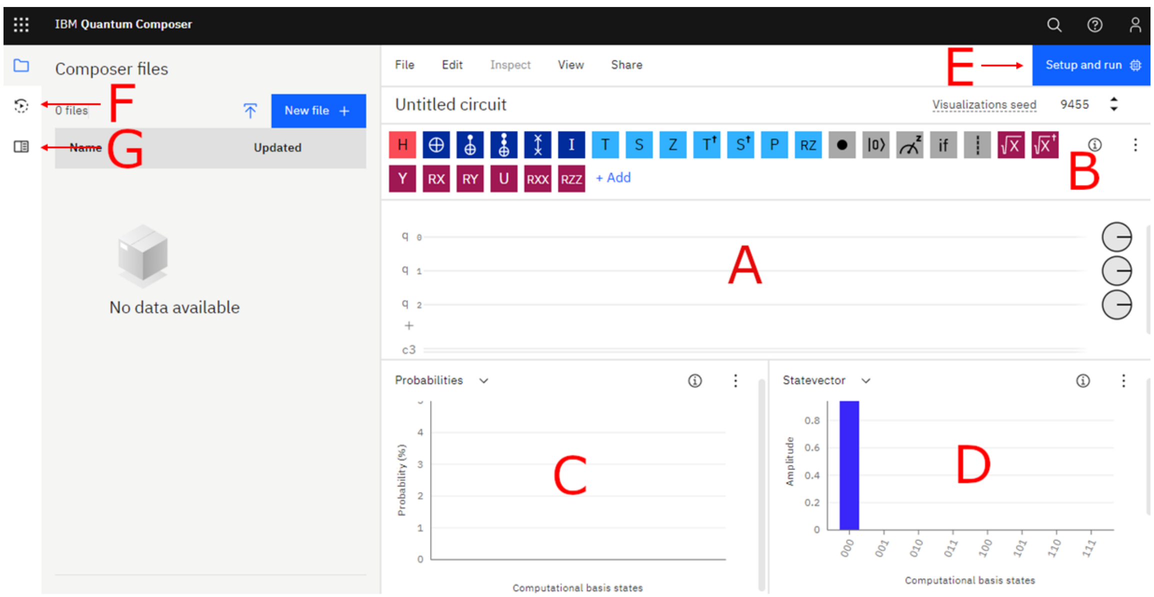


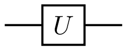
Figure 1: The Quantum Composer user interface. A: The quantum circuit in which gates will be placed; B: Menu with gates available in the quantum composer; C: The simulated results from running the quantum circuit; D: The simulated statevector of all the qubits after running the quantum circuit; E: Button to run the circuit on actual IBM quantum hardware; F: Job status and results; G: Documentation and tutorials by IBM.

The representation of qubits and gates in Figure 1(A) is usually referred to as the *quantum circuit*. The qubits are represented by the horizontal lines labeled $q[0]$, $q[1]$ and so on. The line labeled $c4$ is a classical bit. In this assignment, we will only perform single-qubit operations on $q[0]$, so please delete all other qubuts by clicking on them, and then clicking the trash can icon.

The Quantum Composer will automatically simulate the quantum circuit every time the circuit is changed. The results of this simulation are visible in windows C and D. Window D shows the final state vector of the qubits after all the gates have been applied. Window C shows the probability of getting specific outcomes in the canonical basis $\{|0\rangle, |1\rangle\}$ (called *computational basis* in the quantum computing language) when the qubits are measured. Initially, the qubit is in the $|0\rangle$ state.

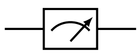
We will be using only the following two quantum gates in this assignment.

- Arbitrary single-qubit gate.

  This gate has three parameters — θ, ϕ and λ — and implements the following evolution operator:
$$\hat{U}(\theta,\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \cdot \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}. \tag{1}$$
  We will set the third parameter, λ, to zero throughout this problem set.

- Measurement.

  Measures the qubit computational basis.

Problem 9. What should the parameters θ and ϕ of the U gate be in order to implement, up to an arbitrary overall phase factor,

a) a half-wave plate at angle α to horizontal (where $|0\rangle = |H\rangle, |1\rangle = |V\rangle$);

b) a quarter-wave plate at angle 0 to horizontal;

c) each of the three Pauli operators?

How would you implement these gates optically if you had a half-wave plate and an arbitrary wave plate with controllable phase shift $\delta\varphi$ between the ordinary and extraordinary axes?

- Place the single-qubit gate $\hat{U}$ onto the rail $q[0]$. Set the parameters of the latter by double-clicking on it: $\theta = \pi/2, \phi = 0, \lambda = 0$. Convince yourself that this circuit will transform the qubit into the state $1/\sqrt{2}(|0\rangle + |1\rangle)$. Check out the state vector in section C of your interface.
- Add the measurement gate to $q[0]$. After this, the state vector data (section C) will no longer be meaningful because a measurement destroys the quantum state.
- Build a quantum circuit with a NOT gate on q0 and a Hadamard gate on q1 and calculate the probabilities of each outcome. Note that adding the 'measurement' gates collapses the quantum state in the Statevector window as well, so the information in that window is no longer useful.
- Click on the 'Setup and run' button (E) in the top right corner of your screen.
- Under 'Step 1', select the quantum computer you would like to use. Preferably choose the one with the fewest number of pending jobs. Some of the options are simulators, and typically have smaller queues. Feel free to use them — but try to run at least a few jobs on a *bona fide* quantum computer.
- Under 'Step 2', keep the number of shots to 1024. This is the number of times your quantum circuit will be executed.
- Under 'Optional', give your job a recognizable name.
- Click the 'Run on [chosen quantum computer]' to execute your quantum circuit.
- You can find the status and results from your job under button F in Figure 1 on the left side of your screen. Your job may take some time to process depending on how busy the quantum computer is.

Problem 10. Construct the circuits to implement single-qubit measurements in

a) canonical;

b) diagonal;

c) circular basis.

Test your circuits by preparing the state $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2\sqrt{2}}(1+i)|1\rangle$ and measuring it in the three bases. Are the resulting measurement statistics consistent with the theoretically expected probabilities?