

## Quantum Club

### Entrance test 2024

1. Point E lies on side BC of a parallelogram ABCD such that  $BE/EC = 2/3$ . F is the intersection point of lines DE and AC. Find the ratio of the areas of triangle AFD and parallelogram ABCD.
2. Find all positive integers  $m, n, p$  such that  $mnp = m + n + p$ .
3. Two tangents of a circle of radius 4 intersect at an angle of magnitude  $2 \arcsin(3/5)$ . Another circle is tangent to both sides of that angle as well as the first circle. Find its radius.
4. A three-digit number is randomly composed of digits 0,3,4,5,6 and 9 such that all digits in the number are different (numbers starting with a zero, e.g. 045, are not considered 3-digit). What is the probability that the number is divisible by 45?
5. Solve the inequality  $\sin 2x + \tan x \geq 2$ .
6. A bottle contains salt dissolved in water. One fourth of the contents is taken from the bottle and heated so that part of the water evaporates and the concentration of salt doubles. The resulting solution is then re-mixed with the contents of the bottle. As a result, the concentration of salt becomes 2% higher compared to the initial solution. Find the concentration of salt in the initial solution.
7. Solve the equation  $\tan 2x + \frac{1}{\sin x} = \frac{1}{\tan x} + \frac{1}{\sin 5x}$ .

Please email your solutions to [Alex.Lvovsky@physics.ox.ac.uk](mailto:Alex.Lvovsky@physics.ox.ac.uk) with the subject "Quantum Club Entrance Test" no later than **noon 15 October 2024**.