Quantum Club

Problems

1. Find $\log_{45}375$ if $\log_3 5 = a$.

2. For which x does the function $y = \sin 2x - 2\sin^2 x$ reach its maximum?

3. Two vectors of length 1 are randomly chosen in a plane. Find the expectation value of the squared length of their sum.

4. A 3-digit number is chosen randomly. What is the probability that all of the following conditions are fulfilled:

   - the number contains only the digits 0,1,4,6,7 and 8;
   - all three digits are different;
   - the number is divisible by 3.

   Note that, e.g., 045 is not considered a 3-digit number.

5. Calculate $\operatorname{tg}\left[\dfrac{1}{4}\left(\arccos\dfrac{3}{5}\right)\right]$.

6. Show that $\sqrt{2\sin\dfrac{3}{4}} < \dfrac{6}{5}$ without using calculators or tables.

7. Find the locus of points z on the complex plane such that $\arg\dfrac{z-1}{z+1} = \dfrac{\pi}{2}$.