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A. I. Lvovsky

Quantum Physics

An Introduction Based on Photons

Springer

A. I. Lvovsky
University of Calgary and Russian
Quantum Center
Calgary, AB
Canada

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Preface

Why I wrote this book

The first rigorous formulation of quantum mechanics (QM) was proposed by Heisenberg and Schrödinger about 80 years ago. Since then, the field has experienced enormous evolution. Initially aimed at explaining atomic spectra, quantum mechanics has now entered the foundation of virtually all branches of physics. Accordingly, QM is an inseparable part of every physics student's training: whatever field future physicists choose after graduation, they will almost certainly need quantum mechanics in their work.

Yet our way of teaching QM to students has not changed much over the years. We begin with the notion of the wavefunction, and write the time-independent, and then the time-dependent Schrödinger equation in the position representation. We determine the energy spectra and the corresponding wavefunctions of simple potential wells, and evolution of wavepackets incident on potential barriers. Finally, we introduce the angular momentum operator and calculate the spectrum of the hydrogen atom. For the last 60 years, this has been, with minor variations, the first semester undergraduate quantum mechanics course program.

This tradition has many advantages. It works with a physical system that a student has already dealt with in classical physics classes, and it is one that they can easily imagine. It allows one to see differences between the behaviors of a classical and a quantum particle, and brings to light several fundamental phenomena that are characteristic of quantumness: tunneling, quantization and the uncertainty principle. It provides a student with the toolbox to solve experimentally relevant problems that cannot be solved classically: after calculating the hydrogen spectrum in the classroom, a student goes to a lab and measures it!

Yet this approach is imperfect. It gives a student an algorithm to analyze a specific physical system, but it does not reveal the inner workings of quantum physics and the logic behind it. We teach our students multiple facts and computational approaches related to wavefunctions, operators, and measurements, but we do not construct a rigorous logical connection among them and do not explain which of these facts are the postulates and which their consequences, and in which logical sequence these consequences are derived.

As a result, the student — at least a *thinking* student — ends up being immensely confused. Why does simply placing hats on top of letters turn a classical expression into a valid quantum one? Why is the momentum operator's action on the wave function equivalent to taking the derivative? Why do we never see momentum eigenstates (and Schrödinger cats) in practice? Why do de Broglie waves have a normalization factor of $1/\sqrt{2\pi\hbar}$? Why do we observe atoms transitioning between energy eigenstates, but not other states? How is a projective measurement related to measuring an observable? Why are some states described by wavefunctions and some by columns of numbers? If all states have norm 1, why don't we normalize the de Broglie wave? If observables are matrices, what is the matrix of the momentum observable?

On top of that there is the most dreaded question. If quantum physics is supposed to be more general than classical, why must one resort to classical notions to understand the concept of measurement? Why is the measurement, in contrast to all other physical processes, not described by unitary evolution? If quantum systems do become classical at some point during the measurement, where exactly is that point?

The fundamental way of thinking we are trying so hard to instill in our students through the years of training in science is "Question everything!". In quantum classes our message seems to be just the opposite: "Shut up and calculate!"[1]

Having been a quantum mechanics student myself, I have eventually found answers to these questions, but in many cases not until long after my PhD. When I asked them as a student, there was no one around, not only to give me the answers, but even to help me state these questions properly.

My quest while writing this book is to address this issue. I attempted to build a clear logical structure, containing as few loopholes as possible. One that would allow the reader to trace each statement down the logical chain back to first principles. One that would *leave no question unanswered*.

So, in a sense, I wrote this book for *myself*. Not for today's myself, but for myself twenty-five years ago. A kind of book that I would have been grateful to have while a third-year student, and one that would have saved me years of agonizing search for the truth.

It is natural to ask: How realistic is my aspiration? Some of the questions I asked earlier sound quite advanced. Perhaps one does need a doctoral degree to answer them?

My answer is twofold. First, there is a pedagogical issue: mechanics, with its Hilbert space of infinite dimension, does not seem to be the best venue for conveying quantum principles. Many of the above questions can be addressed by exemplifying QM with a simpler physical system; I will further elaborate on this below. Second, most of the inconsistencies and paradoxes can be eliminated by properly introducing the notion of entanglement. This notion underlies two essential, mutually rela-

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[1] More on this slogan, incorrectly ascribed to Feynman, in Sec. 2.4.

ted concepts: the *von Neumann measurement* and *decoherence*. The first provides a way to avoid making measurement an exception in the domain of quantum physics, thereby eliminating the Klein bottle logic characteristic of the Copenhagen interpretation. The second describes "inadvertent" measurements that occur naturally, making the quantum world appear to macroscopic observers such as ourselves in the way that is familiar to us under the name of classical physics.

These concepts are not excessively complicated. Mathematically, they are much simpler than many elements of the traditional quantum course, such as the aforementioned hydrogen atom or scattering theory. The main challenge in understanding entanglement is not the challenge to a student's mathematical skills; it rather concerns their imagination. And developing a strong imagination is inherent to becoming a good physicist; as Einstein said, imagination is in fact more important than knowledge.

Quantum mechanics or quantum optics?

The name of our discipline — quantum mechanics – implies that we are studying the applications of quantum principles to the laws of motion. But in fact the framework of quantum theory is not limited to mechanics; it actually applies to all fields of physics. If our aim is to study the general principles of quantum physics, is it wise to choose mechanics as the physical system for illustrating these principles?

Faced with this question, we are compelled to admit that the answer is negative. Mechanics is there mainly due to tradition, because historically, the first successful application of these principles in their modern form was in mechanics. But educationally speaking, using the example of mechanics to explain basic quantum principles is a recipe for disaster. The Hilbert space associated with this system is of infinite dimension; moreover, its basis has the cardinality of the continuum. The student must deal with the unfamiliar, enormously complicated, and not always rigorous mathematical background which includes generalized functions, Fourier transformations, and functional analysis. As a result, instead of concentrating our students' efforts on understanding the physical concepts, we force them to struggle with mathematics, and this often leads to confusion between the end and the means. It is unrealistic to expect any kind of deep understanding to result from this experience. The student simply won't see the forest behind the trees.

If we are to choose the physical system to illustrate quantum physics, we should pick one whose Hilbert space has the lowest nontrivial dimension: two. There is a variety of such systems that are currently studied in the context of quantum information technology as quantum bits. Among them, one stands out as the most thoroughly studied and intuitive: the polarization of the photon. Optical wave polarization would normally already have been studied by the student entering a quantum class. The Jones polarization vectors directly translate into photon polarization state vectors, and the matrices describing the transformation of these vectors by waveplates translate into operators. It is straightforward to argument the measurement postulate from the classical polarization measurement picture, taking into account the discrete nature of the photon. In this way, the quantum fundamentals arise from classical polarization optics (and the students' laboratory experience with the same) in the most straightforward and natural fashion.

Photon polarization is of further benefit when we go on to study entanglement. A vast body of proof-of-principle experiments in quantum information have been performed using this system as the carrier of the quantum bit. Some of these experiments, such as those on quantum cryptography, teleportation and noonlocality, relate directly to the concepts covered in this book. By illustrating the theoretical material with these experiments, right from the start, this book will take students straight into the very heart of quantum physics using examples from today's hottest research topics. And what could make learning an academic discipline more exciting than fresh results from a research lab?

Talking about labs, the student's experience does not have to be limited by reading about experiments done by somebody else. A great advantage of the polarization qubit as the example system is that it is perfectly realistic to augment the course with a laboratory component. Almost all the material of the first chapter is illustrated by a classical polarization experiment containing a laser, a few waveplates, a polarizing beam splitter and two detectors. The material on entanglement can be visualized by a series of labs on single-photon interference remote state preparation and Bell nonlocality. Such experiments are more difficult to set up, but fully within the capabilities of an average physics department, as evidenced by the experience of many colleges around the world, including my home, the University of Calgary. More details on possible educational labs can be found at the book's web site.

The connection between quantum physics and optics in this book is not limited to using the photon to illustrate the discipline's primary concepts. It also manifests itself in the many optical examples scattered throughout the book, as well as the set of subjects chosen for more advanced sections (deep study of the harmonic oscillator, Heisenberg picture, squeezing, density matrices, two-level systems, quantum tomography). These subjects are particularly relevant for those who are interested in quantum optics in particular and quantum information in general.

Structure of the course

The book contains material that can be taught during a two-semester undergraduate quantum mechanics course. In the *first* chapter, the main principles and postulates of QM are introduced and illustrated by the photon polarization qubit. The reader may wish to study this chapter in parallel with Appendix A, which covers the basic linear algebra that is relevant to QM, as summarized in the following table.

Linear algebra concept (Appendix A)	**Quantum concept**	**Physical illustration**
Linear space, basis, dimension, inner product	Quantum state, Hilbert space	Polarization of the photon (Appendix C)
Orthonormal basis	Projective measurement, quantum tomography	Polarization measurements, polarization state tomography, quantum cryptography
Linear operator, Hermitian operator	Observable, uncertainty principle	Pauli matrices as observables in the polarization space
Unitary operator, functions of operators	Schrödinger evolution	Evolution of the photon in a birefringent medium

The *second* chapter is entirely dedicated to entanglement, its consequences and applications. I first introduce the tensor product space mathematically, then explain partial quantum measurements, remote state preparation, and the nonlocality paradox (both the Bell and Greenberger-Horne-Zeilinger forms of it), illustrating the theory with experiments on entangled photons. Nonlocality is arguably the primary paradox of quantum mechanics, and it is natural to follow up with a discussion of the mechanism of quantum measurements, their natural counterpart (decoherence), and the interpretations of quantum mechanics. This section (Sec. 2.4) is where we find out when and why a quantum system becomes classical during a measurement, and why we don't see Schrödinger cats walking around town. Subsequently, I talk in a fairly rigorous fashion about applications of entanglement, such as quantum computation, teleportation and repeaters. When this material is presented in a classroom setting, it is useful to ask two or three students to give presentations on recent experimental research on the subject.

The *third* and fourth chapters are, to some extent, a tribute to the "mainstream" undergraduate quantum mechanics of a particle in a potential field. Here we have to deal with the Hilbert space whose basis is a continuum, so the third chapter is accompanied by a tutorial on Dirac delta functions and the Fourier transform (Appendix D). It is my hope that at this point, when students have already internalized the primary tenets of QM, they will be able to face the technical issues associated with continuous-variable Hilbert spaces without losing sight of the physical principles. As an introduction to continuous-variable systems, I explain how and why some of the normalization rules are affected. Then I present the usual scenarios of potential wells, potential barriers, tunneling, and the harmonic oscillator. This is the point where I envision the first semester to be concluded.

The third chapter goes on to explain the Heisenberg picture and how it is consistent with the Schrödinger picture, illustrating with many examples relating to the physics of the harmonic oscillator (and demonstrated in quantum optics experiments): displacement, phase shift, as well as single- and dual-mode squeezing. With the help of the latter, I present the original version of the Einstein-Podolsky-Rosen paradox.

In the *fourth* chapter, I introduce the three-dimensional geometric space (as a tensor product of three one-dimensional spaces) and explain the angular momentum, spin, and, finally, the hydrogen atom. Then I discuss the behavior of a spin in a magnetic field and magnetic resonance, covering the phenomena of spin echo and Ramsey spectroscopy.

The *fifth* chapter revisits the fundamental principles of QM, now presenting them in the language of density operators, which is of primary significance for all applications of quantum physics. To demonstrate the utility of this language, I apply it to give a formal description of decoherence and relaxation in nuclear magnetic resonance. I then cover the topics that are relevant to modern quantum information science: generalized measurements (POVMs) as well as quantum state, process and detector tomography.

How to use this book (a message to the student)

I have been involved in education, on both sides of the podium, for most of my conscious life. This experience taught me one truth: it is almost impossible to learn anything by passively listening to a lecturer or reading a book. Learning requires active engagement. In the case of theoretical physics, this means that you should perform the derivations yourself rather than observing them performed by somebody else on the blackboard or in the textbook.

With this in mind, I tried to write this text using the Socratic approach: the student comes to the truth by answering the teacher's questions. My own experience with this method comes from high school. I was fortunate enough to attend one of the best Russian science high schools, which has a unique approach to teaching mathematics. Instead of explanations, we were provided with assignments consisting of only of definitions, axioms, and problems. After solving them, we discussed our solution with a tutor, whose task was to ensure that we understood the material correctly.

This book works in the same fashion. You will notice that it contains an unusually large number of exercises. Some of them are conceptual theorems; others are there just for practice; quite a few are both. The idea is that, by solving them one-by-one, you yourself will construct QM with as little help from me as possible. Accordingly, it is not advisable to skip the exercises. This is equivalent to skipping a page or two from a regular textbook: you will not be able to follow what comes next.

Almost all of the exercises have solutions, which are available for download from my website, accessible via `https://www.springer.com/gp/book/9783662565827`. However, please do not look at the solution until you have at least tried to solve the exercise independently. Even if you fail to arrive at the answer yourself, you will be conscious of the point where you are stuck, so the solution will be there as an answer to your existing question. In this way, the seed will fall onto soil that is already fertilized.

On the other hand, I advise that you do have a quick look at the solution even if you have found your own. In this way you will become aware of the errors you (or I) may have made, or of a possible alternative approach to solving the same problem.

Those exercises I consider more difficult are marked with asterisks (*). Unfortunately, many of them contain statements that are essential for subsequent material. Therefore, while it may be acceptable to defer solving (or studying the solutions of) these exercises to the future, you should at least make sure you understand the statements contained therein.

Similarly, sections labeled with asterisks contain more advanced material. You can safely skip these sections without fear to compromise your ability to understand the subsequent content.

Some of the exercises (marked with the symbol §) are not provided with solutions. Usually this is the done when I believe that the problem is relatively simple; in these cases I generally provide an answer immediately after the exercise. Very rarely, there will be an exercise that is marked by both an asterisk and a paragraph sign. These constitute independent research projects that may be worth looking at in your free time.

What knowledge do I expect you to have before you open this textbook?

- You should be familiar with trigonometry (e.g. how to expand $\cos(\alpha+\beta)$ or $\cos\alpha\cos\beta$ into a sum of terms).
- You should be able to work with complex numbers, being familiar with the notions of conjugation, complex phase, and complex exponent (e.g. simplify $|1+e^{i\phi}|^2$).
- You should be reasonably comfortable with probability theory. To help you, Appendix B contains some rudiments of this field.
- The same applies to the physics of optical wave polarization: Appendix C briefly covers the necessary knowledge, but would not be a good replacement for a proper textbook.
- Calculus and ordinary differential equations are essential throughout book, especially Chap. 3 (quantum physics of continuous-variable systems); this requirement extends to multivariate calculus (Jacobian determinant, etc.) for Chap. 4. There is no appendix on calculus, but Appendix D covers the Dirac delta function as well as the direct and inverse Fourier transforms, so pre-existing knowledge of mathematical physics is not required.
- Of primary importance to quantum physics is linear algebra, including the notions of linear spaces, basis, dimension, inner product, orthonormal basis, linear operators and matrices, spectral theorem, functions of operators, and so on. These are covered in Appendix A. However, basic matrix manipulation techniques, such as matrix multiplication, finding eigenvectors and eigenvalues, do not feature in this appendix and should be familiar to you before you start the course.

Acknowledgements

It took me thirteen years to write this book, from January 2005 to December 2017. It is common to remember the end date precisely, because this is the publisher's deadline (in the present case, moved many times, and for a few years). The reason I also remember the start date is that it coincides with the semester I started to teach PHYS 443 Quantum Mechanics I at the University of Calgary. I had just joined the University of Calgary faculty at that time and was not even supposed to teach in that semester. However, when the department head Bart Hicks came to me one day and asked in a nice voice: "Alex, would you like to start teaching a bit earlier? I heard you were interested in Quantum, and we just have a slot", I (a naïve romantic rookie professor) said yes. This is when the first handwritten set of notes was generated.

But the true history of a fruit starts from the roots. And since this book is all about roots, I should also follow this principle in this section. I can trace the roots back to 1962, when my parents, Isay and Tatyana, just a few months before meeting each other in Moscow, saw *Nine Days in One Year* — a Soviet movie about physicists that was cult at that time (by the way, you should see it when you get a chance. It is easy to find online with English subtitles and still a pleasure to watch. And it does profess good values). The cult quickly subsided, but not with my parents. So my profession was decided upon eleven years before I was born. The only disagreement between my parents was whether I should become a member of the Academy of Sciences (the Soviet equivalent of a fellow of the Royal Society) or a Nobel Prize winner. My grandfather reconciled them by pointing out that one does not have to preclude the other.

Fortunately my natural inclinations turned out consistent with my parents' aspirations — in direction if not in magnitude (I sometimes ask myself who I might have become if I had been raised by a different family. I think, either a car mechanic or a software engineer. So experimental physicist seems like a nice compromise). So, skipping a few years, I found myself to be a student of the famous Moscow High School Number 57 (schools in the Soviet Union had numbers, not names) with enhanced training in math and physics. This is where I experienced the Socratic principle of teaching science that I described in the Preface and upon which this book is based. The method was invented by a Moscow teacher Nikolay Nikolaevich

Konstantinov, but the person who taught our class and introduced me to this method was Boris Mikhailovich Davidovich. The storyline of the first two sections from Appendix A and a few exercises from these sections come straight from my high school archives.

Then, college. The professor who introduced me to quantum physics, and fascinated me by it, was Yury Mikhailovich Belousov. He skillfully combined the rigor of the “old school” of Landau and Lifshitz with a colorful, insightful, and passionate teaching manner: “What is the state? An undefinable notion! Like in geometry: you don’t define what a point or a straight line is, do you? State, same thing. What is your state? Drunk? Sober? Tired? That’s the state. The set of states is called the state space. Again, why not? But then we say that this space is linear. Now that’s quite a claim...”

Yet, as also mentioned previously, not all my questions were answered (nor even properly asked) at college, and I had to seek the answers myself for a long time after graduation. On this quest, I have been supported by many brilliant scientists. Just to name a few: Alain Aspect, Konrad Banaszek, Mauro D’Ariano, Hauke Hansen, Peter Marzlin, Philippe Grangier, Miklos Gyulassy, Paul Kwiat, Misha Lukin, Eugene Polzik, Mike Raymer, Barry Sanders, Christoph Simon, Aephraim Steinberg, Ian Walmsley, Xing Wei, and Anton Zeilinger. Two names I should mention separately: my undergraduate advisor, Anatoly Viktorovich Masalov, who introduced me to research, and my PhD advisor, Sven Hartmann, Mr. Photon Echo. In addition to a great deal of science, Sven taught me how to write scientific texts. If this book has any style, it is thanks to him.

While it is difficult to identify a single person who was most instrumental in forming my understanding of quantum physics, I can name a period in my life during which I made most progress. It is the time when I worked as a postdoc at the University of Konstanz, at the institute headed by Dr. Jürgen Mlynek. At that time, the institute was a “Mecca” for quantum physicists, visited by the top brains of the field. Sometimes I was able to steal a few minutes from their busy schedules to discuss those questions I was concerned about, including those of quantum fundamentals (whenever I could collect enough courage to overcome the fear of appearing foolish or ignorant).

Let me now turn back to the moment I started teaching Quantum I in Calgary and wrote my first set of notes. The notes have been rewritten and augmented dozens of times. Perhaps a turning point in converting them into a book was the addition of solutions to the exercises. Initially, solutions were not there; I simply presented them orally during the lectures (I still wonder how these students managed to pass the tests). But then, two important conversations happened. First, I spoke to Jeff Shapiro, an MIT professor who taught me quite a bit of quantum optics during our (alas, brief) meetings. I told Jeff about my idea to convert my lecture notes into a book and about the Socratic method. Jeff looked at me seriously and asked: ”But the problems will have solutions... Right?” And again, almost by a miracle, around the same time, two of my undergraduates, Geoff Campbell and Dallas Hoffman, approached me. “Your notes could really benefit from solutions. We thought, perhaps we can write some of them.” And they did — quite a few of the solutions for the exercises from Chapters 1, 2 and Appendix A come from them, and I am immensely grateful to these guys.

The students' support was in fact paramount at all stages of this book's construction. Since 2005, I taught Quantum I six times to about 200 students, and many of them made important contributions: Russel Bate, Dante Bencivenga, Travis Brannan, Arthur Bury-Jones, Aveek Chandra, Jose da Costa, Ish Dhand, Stefan Donsa, Mark Girard, Chris Healey, Katanya Kuntz, Kimberley Owen, Adarsh Prasad, Mathew Richards, Steven Rogowski, Matthew Townley-Smith, Raju Valivarthi. Their help consisted not only in contributing solutions, but also in finding errors and in asking multiple questions that allowed me to understand which passages were unclear and needed to be improved. Again, I will not be able to name every student who helped, so I have to ask for the kind forgiveness of those who have not been mentioned.

As these notes have been inspired by my own high school experience, I wanted for a long time to try them in the same setting. I was able to realize this in 2013, when I took a sabbatical leave from the University of Calgary to help setting up the Russian Quantum Center in Moscow. At that time, I organized a voluntary quantum physics class — *Kruzhok* or "little circle" in Russian — for Moscow high school students. Together with a team of enthusiast tutors, led by Aleksey Fedorov, we met with the students weekly to hear how they solved the problems in these notes (no solution manual was provided), correct them, explain the subtleties, and — last but not least — discuss the notes themselves. The feedback received in these discussions has been instrumental in shaping up this text, and several members of the Kruzhok — including Aleksey — have now become professional scientists pursuing research on quantum technology on a full-time basis.

I would like to thank Stephen Lyle for thorough proof-reading of the book and providing many insightful comments.

But the greatest thanks of all goes to my wife, Bhavya Rawal. As I am writing these lines, she is driving to pick up our daughter, Sophie, from her grandfather. This is just one of many hundreds of occasions when I really should have been together with my family instead of hiding behind the monitor typing away strange wiggles. But now it seems that even Bhavya's infinite patience wears thin. Yesterday she showed me the movie *Paris Can Wait*, in which the wife of a guy who works too much lets herself get seduced by his French co-worker. Darling, I get the hint. Paris can wait no longer. This is the last sentence I will add to the book!

Calgary, December 10, 2017

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