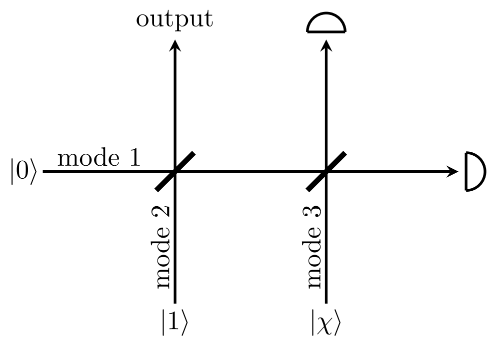
University of Oxford

Department of Physics
Quantum Optics graduate class

Michaelmas term test

November 30, 2021, 9:30–11:30

Problem 1. (50 marks)
The optical scheme shown below, with two symmetric beam splitters, is known as "quantum scissors".



a) (15 marks) The input state is $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$ in parts (a–c) and (e). Find the state of the optical modes 1, 2 and 3 prior to the measurement.

b) (5 marks) Find the state of mode 2 if the detector in mode 1 detects one photon, and the detector in mode 3 detects no photons. What is the probability p of this event?

c) (5 marks) What quantum communication protocol does this scheme represent? Identify the location of the parties and other main elements of this protocol in the above scheme.

d) (5 marks) What is the output if the input state is an arbitrary superposition $|\chi\rangle = \sum_{i=0}^{\infty} \psi_n |n\rangle$? Can you guess why the scheme is called "quantum scissors"?

e) (20 marks) Repeat part (b) if the initial single photon in mode 2 is imperfect and produces a mixture $\hat{\rho}_1 = \eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0|$. The input state is $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$.

You can use the beam splitter evolution operator $|U\rangle$ in the Fock basis as derived in class:

$$\hat{U}|00\rangle = |00\rangle;$$
$$\hat{U}|10\rangle = \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle;$$
$$\hat{U}|01\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle;$$
$$\hat{U}|11\rangle = -\frac{1}{\sqrt{2}}|20\rangle + \frac{1}{\sqrt{2}}|02\rangle,$$

where the operator of the first beam splitter acts on modes (1,2) and the second one on modes (1,3).

Problem 2. (50 marks) A general waveplate with its optical axis oriented at angle θ transforms the polarization state of a photon as follows:

$$\hat{U}|\theta\rangle = e^{i\Delta\varphi}|\theta\rangle; \tag{1}$$
$$\hat{U}\left|\frac{\pi}{2}+\theta\right\rangle = \left|\frac{\pi}{2}+\theta\right\rangle, \tag{2}$$

where $\hat{U}$ is the evolution operator and $\Delta\varphi$ is the difference of the phase shifts of the extraordinary ($|\theta\rangle$) and ordinary ($\left|\frac{\pi}{2}+\theta\right\rangle$) photons.

a) (5 marks) Write down $\hat{U}$ explicitly in its eigenbasis.

b) (5 marks) Suppose this evolution occurs under the action of a fictitious Hamiltonian $\hat{H}$ for the time period t_0. Write down $\hat{H}$ in its eigenbasis.

c) (20 marks) Find the differential equations for the evolution of the three Pauli operators under this Hamiltonian in the Heisenberg picture.

d) (15 marks) Specializing to a quarter-wave plate at $\theta = \frac{\pi}{4}$, solve these equations and find the corresponding evolution of the Pauli operators.

e) (5 marks) Interpret your results: is the evolution consistent with the known properties of waveplates?