University of Oxford

Department of Physics
Quantum Optics graduate class

Michaelmas term test

December 2, 2019

Solve any two of the three problems below.

Problem 1. Consider the evolution of a two-level atom in the rotating-wave approximation in a resonant field with the (real) Rabi frequency Ω. Initially, the atom is in the state $|\psi(0)\rangle$ whose Bloch vector has length 1 and direction between the positive x and z axes, at the angle θ to the z axis. The excited and ground states $|a\rangle$ and $|b\rangle$ are located at the top and bottom of the Bloch sphere, respectively; relaxation is negligible.

a) Write the evolution of the Pauli operators $\hat{\sigma}_{x,y,z}$ in the Heisenberg picture. Find their mean values as a function of time.

b) Find the evolution $|\psi(t)\rangle$ of the atom's state in the Schrödinger picture. Use it to find $\langle\sigma_z(t)\rangle$. Is your result consistent with part (a)?

c) Find the trajectory on the Bloch sphere corresponding to this evolution.

Problem 2. Consider single-mode squeezing defined by the operator $\hat{S}(r)$ with the squeezing parameter r.

a) Show that the single-photon subtracted squeezed-vacuum state $\hat{a}\hat{S}(r)|0\rangle$ equals the squeezed single-photon state $\hat{S}(r)|1\rangle$ up to a normalization factor. Find that factor.

b) Sketch the Wigner function of this state (no calculations are required).

c) Find the mean photon number of this state.

Problem 3. In a cavity quantum electrodynamics setting, the optical field inside a high-finesse cavity interacts with a single atom. When the cavity mode is far detuned from the atomic transition and relaxation is neglected, the physics can be approximated by the Hamiltonian $\hat{H} = \hbar\Omega\hat{n}\hat{\sigma}_z$, where n is the number of photons in the optical mode and $\sigma_z = |a\rangle\langle a| - |b\rangle\langle b|$. Initially ($t = 0$), the atom is in the state $(|a\rangle + |b\rangle)/\sqrt{2}$ while the light mode is in the coherent state $|\alpha\rangle$ with a real and positive amplitude α.

a) Write the state of the light-atom system at arbitrary time t (the optical mode can be represented in the photon number basis). At which time does this state become equal to the initial?

b) At the moment $t = \pi/2\omega$, the atom is measured in the basis $\{(|a\rangle \pm |b\rangle)/\sqrt{2}\}$. This measurement prepares the light mode in a certain pure state dependent on the measurement result. Write the wavefunction of this state in the momentum basis.

   Hint: answering this question does not require any complicated calculations.

c) Write the density matrix $\hat{\rho}_l(t)$, with $t = \pi/\omega$, of the light mode alone if no information about the atom is available.

d) Answer the same question about the density matrix $\hat{\rho}_a(t)$ of the atom.