University of Oxford

Department of Physics
Subdepartment of Atomic and Laser Physics
Graduate class 2018

Michaelmas term test

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Problem 1. N photons are initially in a GHZ-like state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|H \dots H\rangle + |V \dots V\rangle \right).$$

a) All but one photons are measured in the diagonal basis. M measurements yielded the result $|+45°\rangle$, the remaining $N - M - 1$ yielded $|-45°\rangle$. Find the probability of this event and the state of the remaining photon.

b) The same question for the measurement in the circular basis. M measurements yielded $|R\rangle$, the remaining $N - M - 1$ yielded $|L\rangle$.

Problem 2. A harmonic oscillator, initially in the vacuum state $|\psi(0)\rangle = |0\rangle$, evolves under the Hamiltonian

$$\hat{H} = -\frac{1}{2}\hbar\gamma(\hat{X}\hat{P} + \hat{P}\hat{X}) + \hbar d\hat{P},$$

where $\gamma, d > 0$.

a) Find the mean and variance of the position and momentum observables as a function of time in the Heisenberg picture.

b) Sketch the contour plot of the Wigner function of the corresponding state $|\psi(t)\rangle$. How would you call this state?

c) Find the Fock decomposition of the state $|\psi(t)\rangle$ up to the first order in time for $\gamma t, dt \ll 1$ using the Schrödinger picture.

d) Find the mean values of the position and momentum observables from the result of part (c) and check consistency with part (a).

Problem 3. A two-level atom, initially in the ground state $|\psi(0)\rangle = |b\rangle$, evolves under a field with the Rabi frequency Ω and detuning Δ. The relaxation constant of the atom is Γ. The field is weak so that either $\Omega \ll \Gamma$ or $\Omega \ll \Delta$.

a) Write the differential equation for the evolution of the state under the rotating wave approximation and the stochastic wavefunction approach.

b) Solve this equation and find the amplitude of the excited state as a function of time.

c) Show the evolution of the state on the Bloch sphere for the cases $\Delta \ll \Gamma$ and $\Gamma \ll \Delta$.