A Short History of Quantum

Axel Kuhn

A Short History of Quantum

- ***What is wrong with “classical” physics?***
  - ➙ *The dawn of quantum physics*
- ***Quantum nature of light***
  - ➙ *particle-like behaviour of waves*
- ***Quantum nature of matter***
  - ➙ *wave-like properties of matter*

} *wave-particle duality*

- ***Does God play dice?***
  - ➙ *Uncertainty principle, superposition, and entanglement*
  - ➙ *Schrödinger‘s cat and other paradoxes*

The Dawn of Quantum Physics

◦ **Classical Physics ➙ Deterministic**

For an intellect which knows all forces that set nature in motion, and all positions of all items of which nature is composed, nothing would be uncertain and the future just like the past would be present before its eyes

Laplace's demon (Pierre-Simon Laplace, 1814)

- ***fails to describe nature correctly***
- ***very intuitive and easy to interpret***

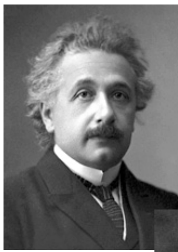
The Dawn of Quantum Physics

◦ **Quantum Physics ➙ *Probabilistic***

- *quantisation of energy (discrete energy levels)*
- *wave-particle duality and interference of particles*
- *uncertainty relation*
- *Schrödinger's cat, superposition, entanglement*

- ***most successful description of nature***
- ***not intuitive and many interpretations***

The Dawn of Quantum Physics



Failures of classical physics
... explained by quantum mechanics

- *photoelectric effect*
  (Albert Einstein, 1905, NP 1921)
- *black-body radiation*
  (Max Planck, 1900, NP 1918)
- *atomic spectra and structure*
  (Niels Bohr, 1913, NP 1922)

The Dawn of Quantum Physics

Failures of classical physics
... continued

- *interference of particles*
  (L. de Broglie, 1924, NP 1929)

Creation of quantum mechanics

- *uncertainty principle*
  (W. Heisenberg, 1927, NP 1932)
- *Schrödinger's wave equation*
  (E. Schrödinger, 1926, NP 1933)

Let there be Light

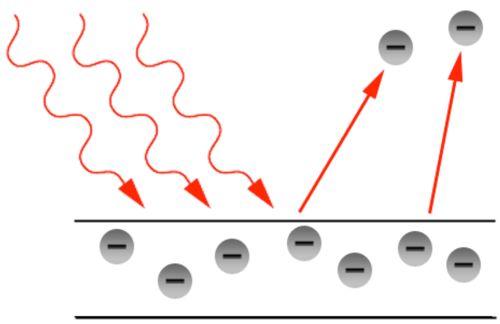
- ***Newton postulates light "particles"***
  - *reflection and refraction*
- ***Interference and diffraction ➙ Waves***
  - *Young's double-slit experiment*
  - *Maxwell's equations ➙ electromagnetic wave*
  - *diffraction, interference*
- ***Quantised radiation ➙ Photons***
  - *Photoelectric effect and black-body radiation*
  - *Compton scattering*

Photoelectric Effect

$$E_{kin} = h\nu - E_{bind}$$

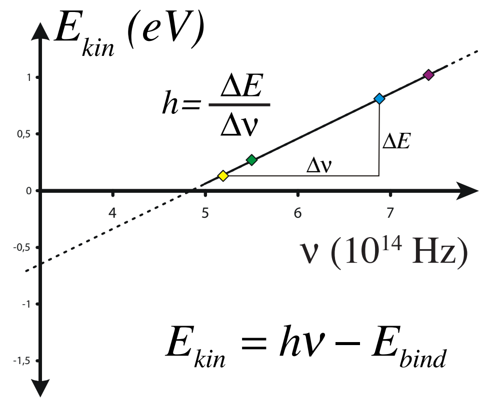
↓

quantised?

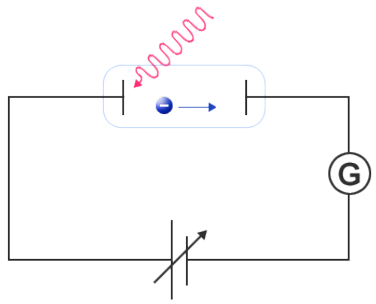


wavelength λ
frequency $\nu = c/\lambda$
photon energy $h\nu = hc/\lambda \quad \Rightarrow$ *Quantisation of Radiation*

Photoelectric Effect



$$E_{kin} = h\nu - E_{bind}$$

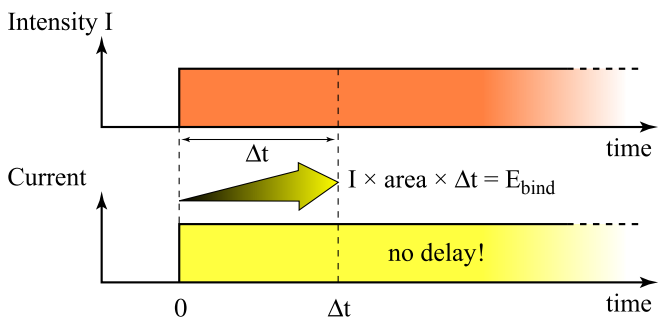


- *different colours (frequencies)*
- *threshold gate voltage = max. kinetic energy*

Planck's constant $h = 6.626 \times 10^{-34}$ *Js*

Photoelectric Effect

- *electron binding energy* $\Rightarrow$ *quantised interaction* $\quad E_{kin} = h\nu - E_{bind}$
- *no delay* $\Rightarrow$ *quantised radiation*



ANNALEN

DER

PHYSIK.

6. *Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt;*
von A. Einstein.

Zwischen den theoretischen Vorstellungen, welche sich die Physiker über die Gase und andere ponderable Körper gebildet haben, und der Maxwellschen Theorie der elektromagnetischen Prozesse im sogenannten leeren Raume besteht ein tiefgreifender formaler Unterschied. Während wir uns
[...]
den Frequenz) keine merkliche nicht von Ionisation begleitete Absorption aufweist.

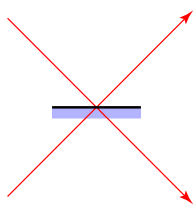
Bern, den 17. März 1905.

1) J. Stark, Die Elektrizität in Gasen p. 57. Leipzig 1902.
2) Im Gasinnern ist die Ionisierungsspannung für negative Ionen allerdings fünfmal größer.

(Eingegangen 18. März 1905.)

So what is a “Photon” ?

- *smallest amount of light of a given frequency*
  - ➙ *a photon was there, when my detector clicks (FAPP)*
  - ➙ *energy quantum of a field mode (Planck)*
- *cannot be divided*
  - ➙ *effect of half-silvered mirrors?*



- *wave- and particle like behaviour*
  - ➙ *(wave amplitude)^2 ⇒ probability of photodetections*
  - ➙ *interference ... only with itself (Dirac)*

Momentum of a Photon

- *quantisation of energy* $E = h\nu = hc/\lambda$
- *for light,* $E = mc^2 = pc$
- *so the photon momentum is*

$$p = h/\lambda = \hbar k$$

Wave-Particle Duality

- *Luis de Broglie, 1924:* ***De-Broglie wavelength***

$$\lambda = h/p \text{ or } p = \hbar k$$

- ***Any particle*** *of momentum* ***p*** *and energy* $E=\hbar\omega$

$$\psi(x,t) \propto \exp[i(kx - \omega t)]$$

with $\hbar=h/2\pi$ *and* $\omega=2\pi\nu$

- *plane wave*
- *not localised*

Uncertainty Principle

Solutions in free space → plane waves $\propto \exp[i(kx - \omega t)]$

- *delocalised:* $P(x)dx = |\psi(x,t)|^2\, dx = const.$
- *not normalised* $\int P(x)dx = \int |\psi(x,t)|^2\, dx = \infty$

$$\Delta p = 0$$
$$\Delta x = \infty$$

Single-slit diffraction in transverse direction

- *slit width* Δx *defines uncertainty in position*
- *first diffraction minimum at* $\dfrac{\lambda}{\Delta x} = \dfrac{h}{p\Delta x} = \dfrac{\Delta p}{p}$

$$\Delta x \Delta p = h$$

Fourier transform between x and k space

- *for a Gaussian of width* Δx*, this leads to* $\Delta x \Delta k \geq 1/2$

$$\Delta x \Delta p \geq \hbar/2$$

Wave Equations from *E=T+V*

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)$$

Time-dependent Schrödinger Equation ***(TDSE)***

V(x) time independent
$\Rightarrow$ *separation of variables*

$E = \hbar\omega$
$\psi(x,t) = \varphi(x)e^{-i\omega t}$

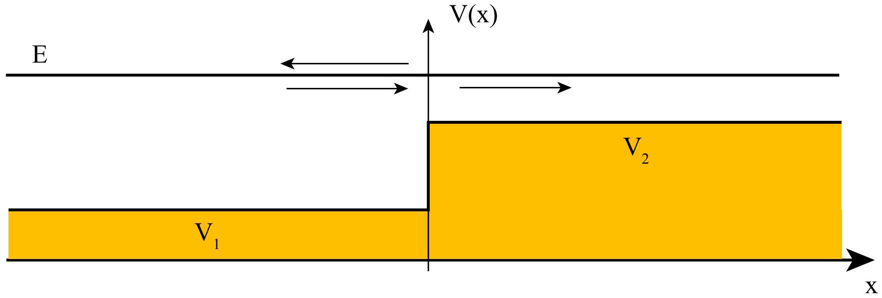
note that $\psi(x) = \langle x | \Psi \rangle$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x) + V(x)\varphi(x) = E\varphi(x)$$

Time-independent Schrödinger Equation ***(TISE)***

Simple Example: Potential Barrier

$$\varphi_1 = Ae^{ik_1x} + Be^{-ik_1x} \qquad \varphi_2 = Ce^{ik_2x} + De^{-ik_2x}$$

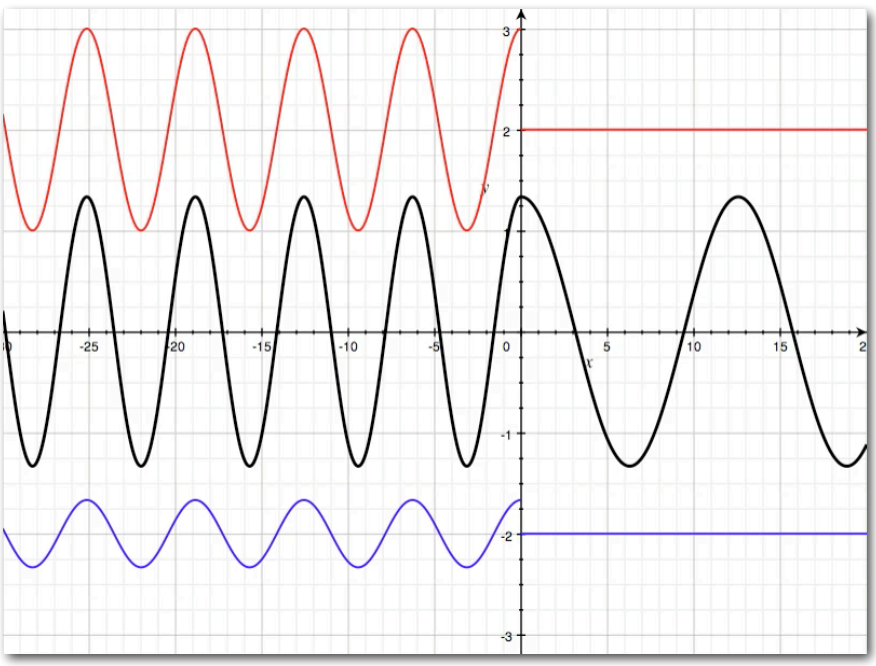


$$\hbar k_1 = \sqrt{2m(E - V_1)} \qquad \hbar k_2 = \sqrt{2m(E - V_2)}$$

Reflection probability

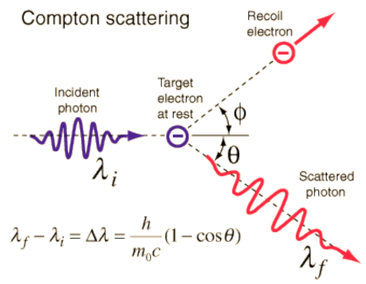
$$R = \left(\frac{B}{A}\right)^2 = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

Potential Barrier



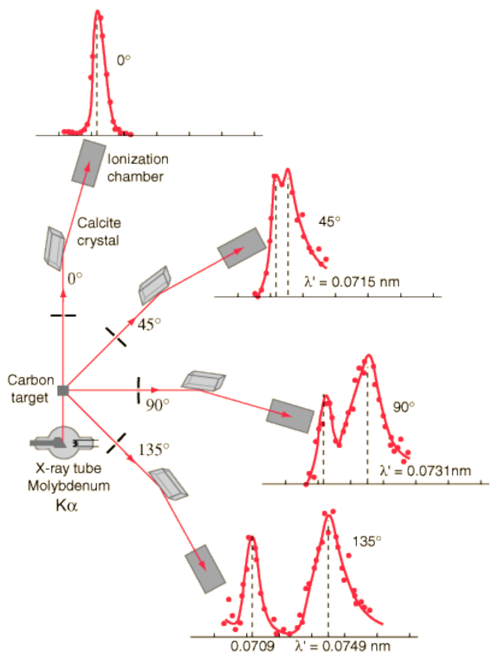
Light & Matter

Compton Scattering



elastic photon-electron scattering

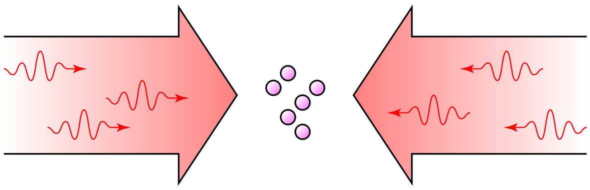
Arthur Compton, 1922



Photon-Particle Collisions

Modern Atomic Physics ➙ *Atom Optics*

- *Laser cooling and trapping of Atoms*
- *Bose-Einstein condensation (BEC)*

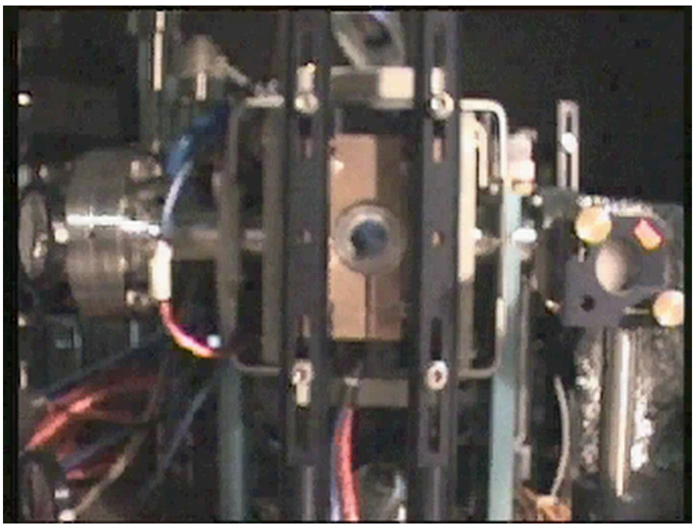


optical molasses

Control and manipulate motional degrees of freedom with light

$$p_f = p_i \pm \hbar k$$

Magneto-Optical Trap

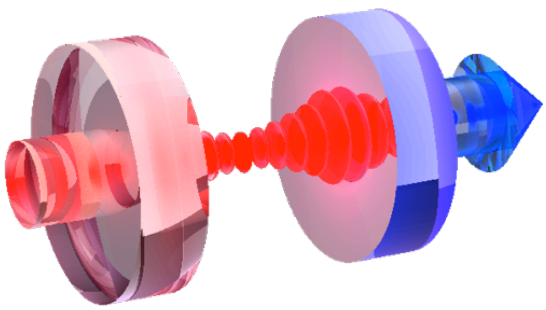


trapping and cooling of Rubidium to $T = 30\,\mu K$

(movie taken by I. Bloch, University of Mainz)

Planck

Photons in a Box



standing wave in a one-dimensional cavity

nodes at the end points

standing wave on a violin string

Black-Body Radiation

- ***Our "box" represents a black body ...***

  in equilibrium it radiates in accordance with its temperature.

- ***The radiated energy ...***

  comes from standing waves in a three-dimensional cavity.

- ***Spectral energy density:***

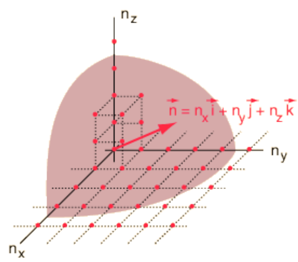
  $\varrho(\nu)\, d\nu = g(\nu)\, d\nu \times \langle E\,(\nu)\rangle =$ *spectral mode density* $\times$ *avg. Energy*

Black-Body Radiation

- ***Hollow cube of edge length L***

resonance condition (3D)

$$n_x^2 + n_y^2 + n_z^2 = \frac{4L^2}{\lambda^2}$$



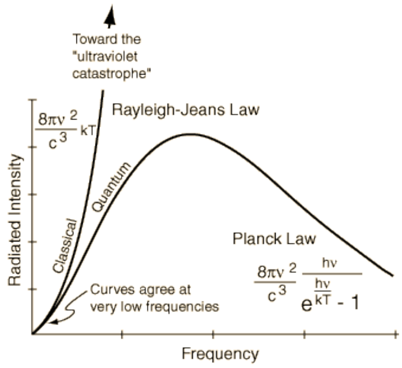
Spectral density of modes:

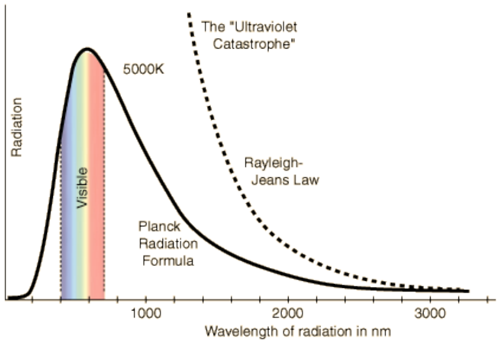
$$g(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu$$

Black-Body Radiation

- ***Rayleigh-Jeans law***
  ➙*ultraviolet catastrophe*

$$\rho(\nu)d\nu = k_B T \times \frac{8\pi\nu^2}{c^3}d\nu$$





- ***Planck's law***

$$\rho(\nu)d\nu = \frac{h\nu}{e^{h\nu/k_BT}-1} \times \frac{8\pi\nu^2}{c^3}d\nu$$

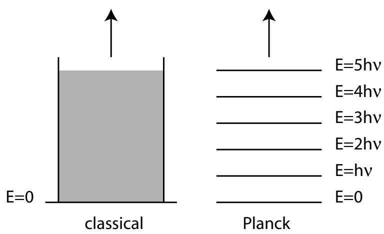
Planck's Postulate

***Any physical quantity** which **oscillates** in time has a total **energy E** which satisfies*

$$E = nh\nu \quad \text{with} \quad n = 0,1,2,3,...$$

ν is the frequency and
h is Planck's constant

$h = 6.626 \times 10^{-34}$ *Js*



Matter Waves

Interference of Matter Waves

Has been shown with

- *electrons*
- *neutrons*
- *atoms*
- *molecules*

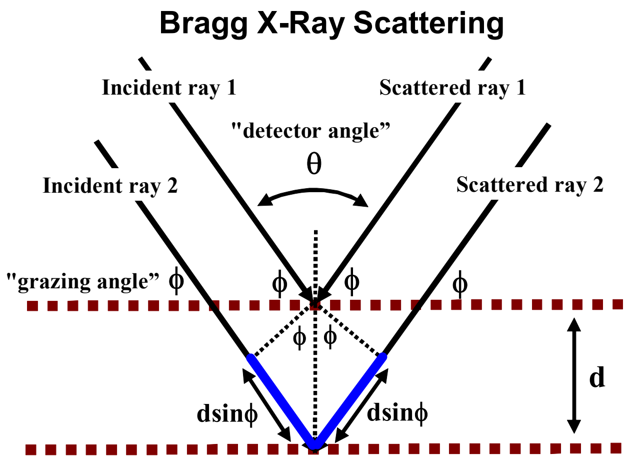
→ *C_{60}, DNA molecules*

- *Bragg scattering*
- *double slit*
- *diffraction grating*
- *light (standing wave)*

Davidson-Germer Experiment

Bragg scattering

$$\Delta r = r_2 - r_1 = 2d\sin\phi$$



$$\sin\phi = m\frac{\lambda}{2d}$$

Davidson-Germer Experiment

Bragg scattering with electrons (1925, NP 1937)

E=54 eV and λ=0.167 nm

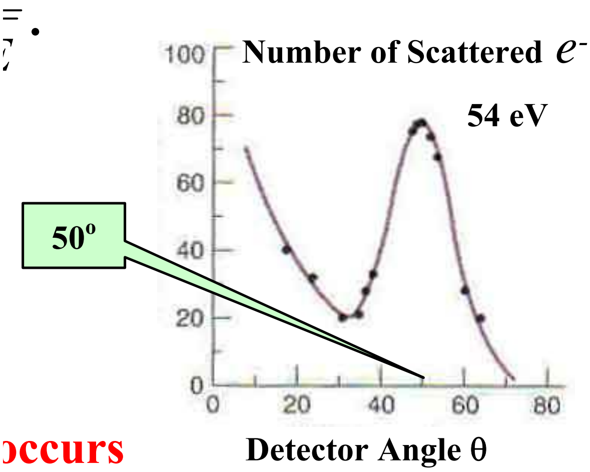
d=0.092 nm (Ni crystal)

$$\sin\phi_1 = \frac{\lambda}{2d} = \frac{0.167nm}{2(0.092nm)} \approx 0.908$$

$$\phi_1 \approx 65^{\circ}$$

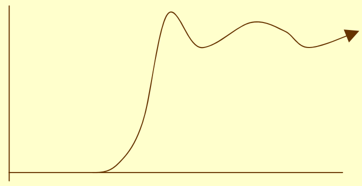
$$\theta_1 = \pi - 2\phi_1 \approx 50^{\circ}$$

occurs

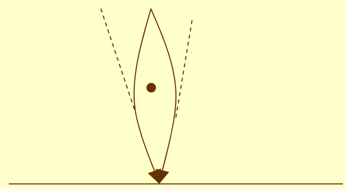


More Experiments with Electrons

a. Diffraction from edge by using a gold wire (Boersch, 1956)



b. Two-Slit Diffraction by using a charged fibre



c. The electron microscope is used in much of materials and biological science and has a resolution of 0.1 nm.

Diffraction of Atoms and Molecules

Double Slit (Carnal & Mlynek, 1991)

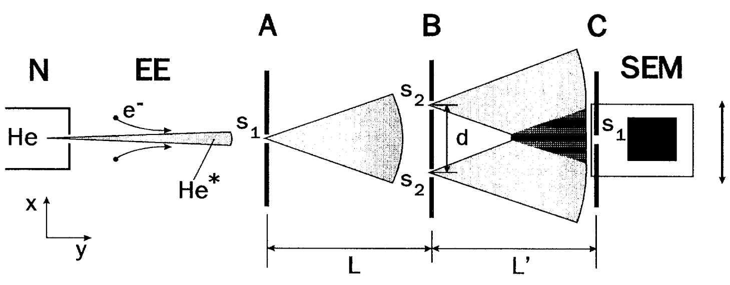


FIG. 2. Schematic representation of the experimental setup: nozzle system and gas reservoir N; electron impact excitation EE; entrance slit A, double slit B, and detector screen C; secondary electron multiplier SEM (mounted together with C on a translation stage). Dimensions: $d = 8\ \mu m$, $L = L' = 64$ cm; slit widths: $s_1 = 2\ \mu m$, $s_2 = 1\ \mu m$.

Diffraction of Atoms and Molecules

Interference pattern
↕
convolution with detector resolution (2μm)

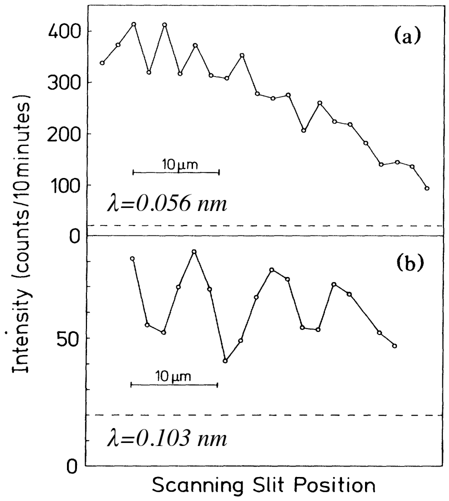


FIG. 4. Measured atomic intensity profiles in the detector plane as a function of the lateral detector position *x*. The profile is probed with the 2-μm-wide single slit. Atomic wavelength (a) λ_{dB}=0.56 Å and (b) λ_{dB}=1.03 Å. The number of

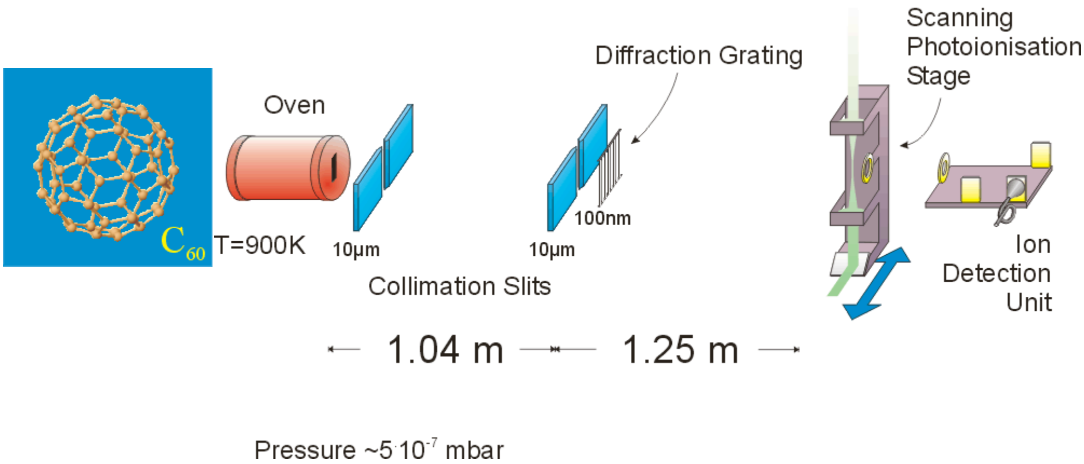
Diffraction of Atoms and Molecules

- *C_{60} (Bucky balls)*

v = 210 m/s; m = 720 amu = 1.2×10^{-24} kg

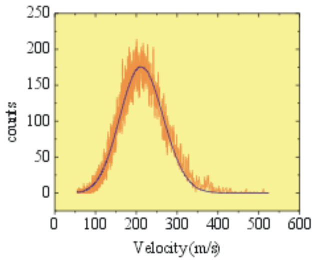
λ_{db} = 2.5 pm = 0.0025 nm

M. Arndt, A. Zeilinger, 1999

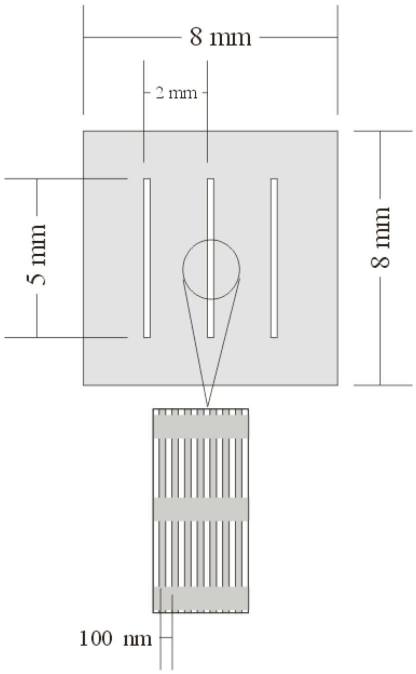


Diffraction of Atoms and Molecules

- *C_{60} (Bucky balls) velocity:*



- *SiN diffraction grating:*
  d = 100 nm
  a = 50 nm



Diffraction of Atoms and Molecules

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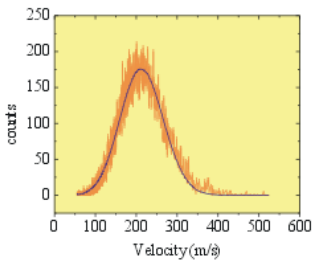

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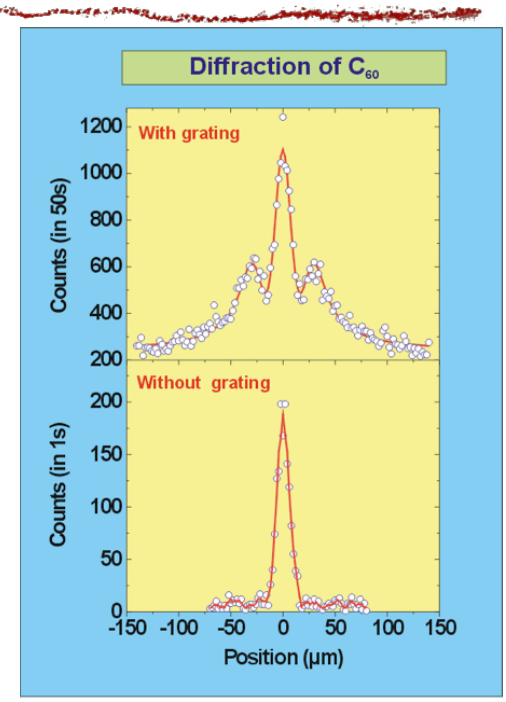
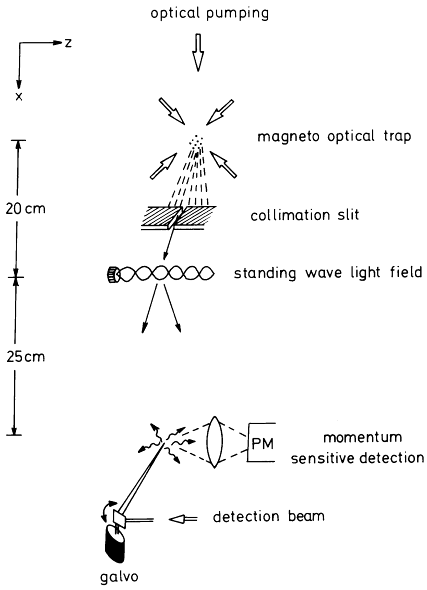


Diffraction of Atoms from a Light Wave

- *standing wave*
- *planes of equal intensity*
- ***short** interaction time:*
  - *diffraction grating*
- ***long** interaction time:*
  - *Bragg scattering*

Pritchard, 1988 (atomic beam)
Rempe, 1996 (MOT)



Diffraction of Atoms from a Light Wave

- *short interaction time*
- *optical grating for atoms*
- *photon exchange with both waves*
- *distribution depends on intensity, detuning and interaction time*

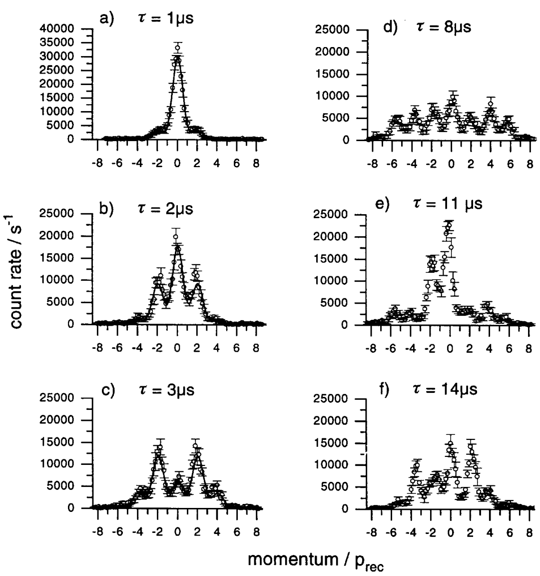
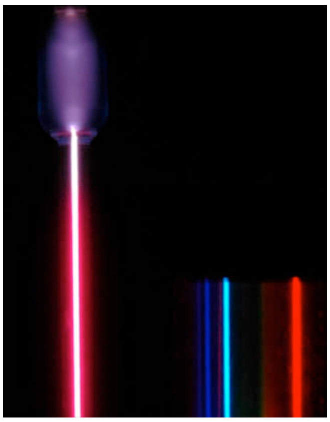


Figure 6. Momentum distribution of atoms after diffraction from a standing light wave in the regime of short and intermediate interaction times. In part (*a*), (*b*), and (*c*) the solid lines represents fits based on the Raman–Nath theory. In part (*d*), (*e*) and (*f*) the Raman–Nath approximation in clearly not valid.

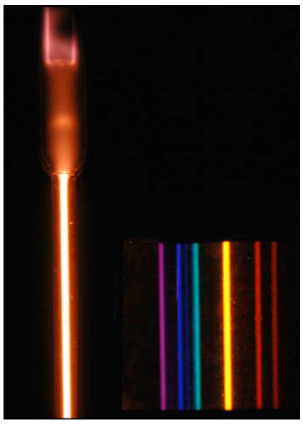
Atoms

Quantised Energy Levels

Atomic spectra (from discharge lamps)



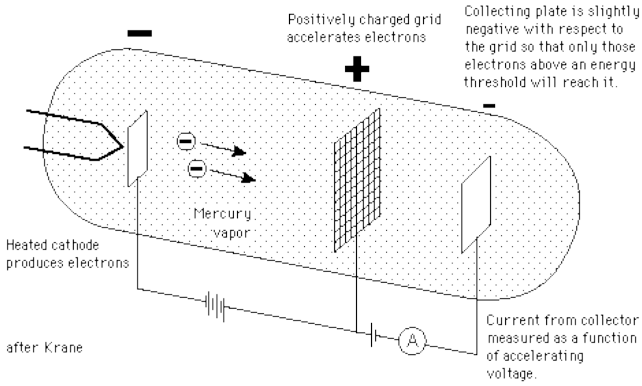
hydrogen atoms



helium atoms

Quantised Energy Levels

Frank-Hertz Experiment (1914, NP 1925)



electrons accelerated in mercury vapour

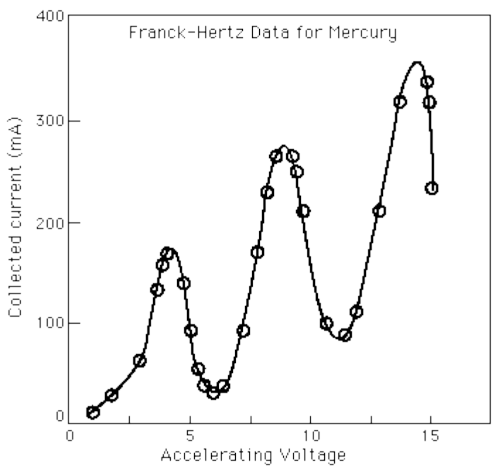
Quantised Energy Levels

Frank-Hertz Experiment (1914, NP 1925)

electron current drops every 4.9V

⇓

inelastic collisions occur whenever $E_{el} = 4.9\ eV$



Bohr's Model

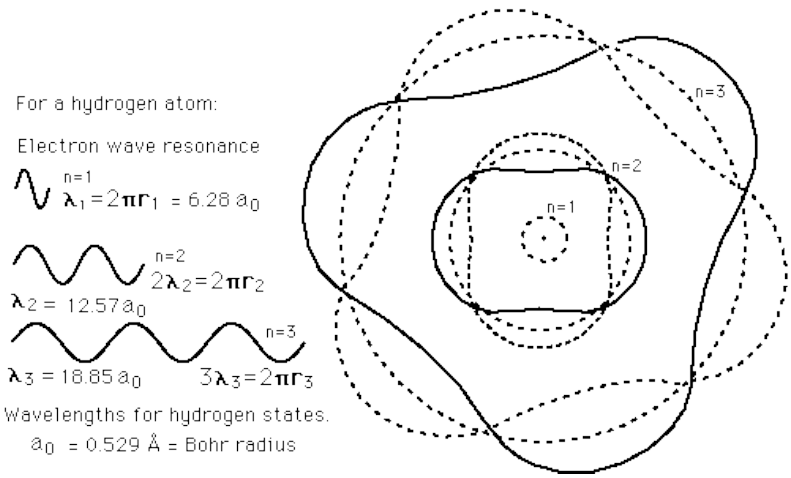
- *classical electron orbit*
- *circumference = integer multiple of λ_{dB}*

$$2\pi r = n\lambda_{dB}$$

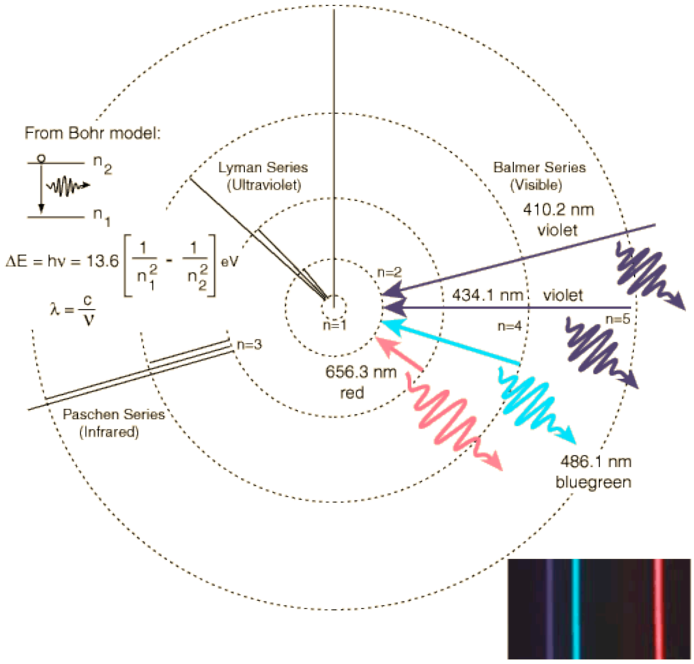
- *quantisation of angular momenta*

$$L = mvr = n\hbar$$

Bohr's Model



The Hydrogen Spectrum

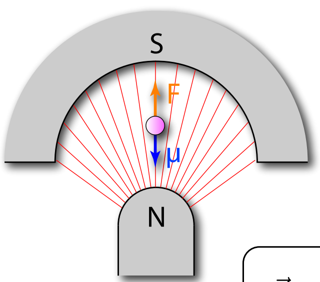


$$E_n = -\frac{Rhc}{n^2}$$

$$Rhc = 13.6\ eV$$

Stern-Gerlach: An Atomic Beam Splitter

- *Dipole in magnetic fields* ➙ *torque* $\quad \vec{\tau} = \vec{\mu} \times \vec{B}$
- *B field gradient* ➙ *net force*

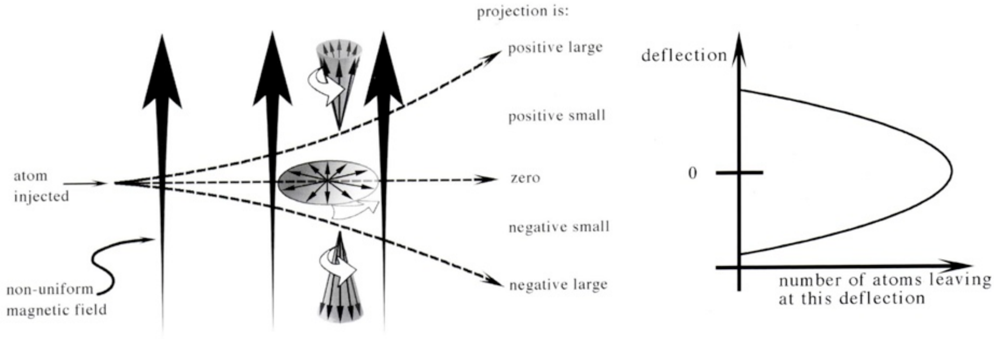


$$V = -\vec{\mu} \cdot \vec{B}$$

$$\vec{F} = -\nabla V = \nabla(\vec{\mu} \cdot \vec{B}) = \sum \mu_i \nabla B_i \longrightarrow \left(\mu_z \frac{d}{dz} B_z\right) \hat{z}$$

Stern-Gerlach Experiment

- *Silver ➙ one valence electron*
  - *expectation: random orientation of μ and B*



Stern-Gerlach Experiment

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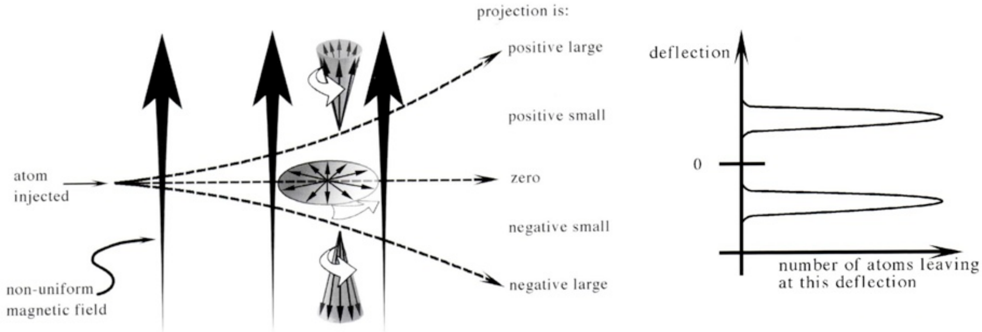

two angles found ⇒ only ***two*** $\boldsymbol{\mu_z}$ ***values***

The End

classical description incomplete

↓

quantum ideas: $\Delta p \Delta x \geq \hbar/2$ $\quad E = \hbar\omega$

$p = \hbar k$ $\quad \lambda = h/p$

↓

success! *Bohr's hydrogen atom* *Matter waves*

Black-body radiation *Photoelectric effect*

↓

formalism $\left(-\frac{\hbar^2}{2m}\Delta + V(\vec{x})\right)\Psi(\vec{x},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t)$ $\quad \langle a|b\rangle = \delta_{ab}$ $\quad H|\psi\rangle = E|\psi\rangle$

$|\psi\rangle = \alpha|a\rangle + \beta|b\rangle$

↓

quantum engineering *Schrödinger's cat, EPR, entanglement,*

Quantum Cryptography & Computing