A13482W1

SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B3: QUANTUM, ATOMIC AND MOLECULAR PHYSICS

TRINITY TERM 2019

Tuesday, 11 June, 2.30 am – 4.30 pm

Answer **two** *questions.*

Start the answer to each question in a **fresh book.**

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. (a) Explain what is meant by the *LS-coupling* scheme. Outline how the residual electrostatic interaction leads to the formation of terms, and explain why these can be labelled by the quantum numbers L and S. [3]

(b) Explain how the spin-orbit interaction leads to fine structure in the LS-coupling scheme. Derive the interval rule for the fine structure splitting between adjacent fine structure levels. [6]

(c) In the emission spectrum of Mg ($Z = 12$), strong spectral lines are observed at wavelengths of 285.21 nm, 516.73 nm, 517.27 nm, 518.36 nm, and 1182.8 nm. The line at 285.21 nm is also observed in absorption. In separate experiments it is established that the singlet–triplet splitting in the 3s4s configuration is 230 590 m^{-1}. Use this information to sketch an energy level diagram for Mg. You may assume that the observed wavelengths arise from the three lowest-lying configurations. Identify the observed transitions and state the energy of each level in units of m^{-1}. Establish whether the interval rule is obeyed in this case. [10]

(d) The table below gives the energy levels for some configurations in the group II element Ba ($Z = 56$). Comment on whether the LS-coupling scheme is a good approximation in this case. [6]

Configuration	Level energies m^{-1}
$6s^2$	0
6s5d	903 397; 921 550; 959 653; 1 139 535
6s6p	1 226 602; 1 263 662; 1 351 474; 1 806 026

2. (a) A solid target is bombarded by an energetic electron beam of kinetic energy E_{e}. What are the processes responsible for the emission of X-rays? Describe the X-ray emission spectrum and how this changes as E_{e} is increased. [5]

(b) The same target is now illuminated by X-rays of energy E_γ. Describe how the X-ray absorption varies as E_γ is increased. Sketch the X-ray emission and absorption spectra as a function of photon energy, indicating clearly any features which link the two spectra. [5]

(c) The X-ray emission spectrum of Ag is recorded using an X-ray tube. Several sets of characteristic X-rays appear as the anode voltage is increased, and a final set of such lines appears at a threshold anode voltage of 25.514 kV. In this final set of characteristic X-rays, the most intense emission occurs on a pair of lines with photon energies of 21.990 keV and 22.163 keV. Explain these observations. Sketch an energy level diagram giving the energies of the levels involved in the emission of this final pair of characteristic X-rays. Label the energy levels with appropriate quantum numbers. [5]

(d) In addition to the emission of characteristic X-rays, electrons are also emitted with energies of 17.793, 18.088, 18.258, 18.331, 18.512, and 18.681 keV. Describe the process responsible for the emission of these electrons. Using the energy levels deduced above, identify the transitions responsible for three of the emitted electron energies. Give the quantum numbers of the level responsible for the other three observed electron energies and estimate its energy. [8]

(e) Give reasons for any differences between the observed energies of the emitted electrons and those you calculated from the spectrum of characteristic X-rays. [2]

3. A two-level atom is illuminated by monochromatic radiation with an electric field which can be written as $\vec{E} = \vec{E}_0 \cos(\omega t)$. The amplitudes $c_2(t)$ and $c_1(t)$ of the upper and lower states respectively are then given by,

$$\frac{\mathrm{d}c_2}{\mathrm{d}t} = -\mathrm{i}\Omega_\mathrm{R} c_1(t) \mathrm{e}^{\mathrm{i}\omega_0 t} \cos(\omega t)$$

$$\frac{\mathrm{d}c_1}{\mathrm{d}t} = -\mathrm{i}\Omega_\mathrm{R} c_2(t) \mathrm{e}^{-\mathrm{i}\omega_0 t} \cos(\omega t),$$

where $\hbar\omega_0 = E_2 - E_1$ is the energy spacing of the two levels.

(a) Give an expression for Ω_R in terms of the dipole matrix element for the two levels, and explain its role in describing the interaction of the atom with the radiation field. You may assume that Ω_R is a real number. [3]

(b) Justify the rotating wave approximation, and use this approximation to show that,

$$\frac{\mathrm{d}^2 c_2}{\mathrm{d}t^2} + \mathrm{i}\delta\omega \frac{\mathrm{d}c_2}{\mathrm{d}t} + \frac{\Omega_\mathrm{R}^2}{4} c_2(t) = 0,$$

where $\delta\omega = \omega - \omega_0$. Assuming the boundary conditions $c_1(0) = 1$ and $c_2(0) = 0$ show that,

$$|c_2(t)|^2 = \left(\frac{\Omega_\mathrm{R}}{\Omega_\mathrm{eff}}\right)^2 \sin^2\left(\frac{\Omega_\mathrm{eff}}{2} t\right)$$

and find an expression for Ω_eff. *Do not assume that the radiation is weak.* [10]

(c) On the same graph, sketch $|c_2(t)|^2$ when $\delta\omega = 0$ and $\delta\omega = \Omega_\mathrm{R}$, indicating clearly the key features. [6]

(d) How does the response of the atom to the radiation field change as the detuning $\delta\omega$ is changed? Without detailed calculation, describe how your results can explain the behaviour of the atom in a weak, broad-band radiation field. [6]

4. (a) Describe what is meant by the term *gain saturation* when applied to a laser amplifier. Explain why gain saturation occurs. [4]

(b) The upper and lower atomic levels of a homogeneously broadened laser amplifier have fluorescence lifetimes τ_2 and τ_1, and they are excited at constant rates R_2 and R_1 per unit volume, where '2' denotes the upper level, and '1' the lower. Assuming that the degeneracies of the two laser levels are equal, show that in the steady-state the population inversion density is given by,

$$N^*(0) = R_2\tau_2\,(1 - A_{21}\tau_1) - R_1\tau_1,$$

where A_{21} is the Einstein A-coefficient of the laser transition. [4]

(c) Show that in the presence of narrow-band radiation of angular frequency ω and total intensity I, the population inversion density becomes,

$$N^*(I) = \frac{N^*(0)}{1 + I/I_s(\omega)},$$

and find an expression for the saturation intensity $I_s(\omega)$ in terms of the optical gain cross-section $\sigma_{21}(\omega)$. Discuss the importance of the parameter $I_s(\omega)$, and explain why it depends on the frequency of the radiation. [10]

(d) An experiment is performed to determine the saturation intensity by recording the intensity leaving the amplifier for incident beams of two different frequencies: $\omega_1 = \omega_0$, where ω_0 is the centre frequency of the laser transition, and $\omega_2 = \omega_0 + \frac{1}{2}\Delta\omega$, where $\Delta\omega$ is the full-width at half maximum linewidth of the laser transition. When the incident beam is tuned to ω_1, the intensity of the output beam is found to be $I_1 = 3.2\,I_0$, but when tuned to ω_2 the output is $I_2 = 2.1\,I_0$. In both cases the incident intensity is I_0. Find the saturation intensities at frequencies ω_1 and ω_2 in terms of I_0. If the length of the gain medium is 50 mm, what is the small-signal gain coefficient in each case? [7]