Problem Set 4

Laser Physics

1. Homogeneous broadening:

   (a) Explain what is meant by the terms **homogeneous broadening** and **inhomogeneous broadening**. Give two examples of each class of broadening.

   (b) Describe in outline how the natural linewidth of a transition is consistent with the Uncertainty Principle. What is the natural linewidth of a transition between two levels with radiative lifetimes of τ_1 and τ_2?

   (c) Show that the full-width at half-maximum linewidth of a Doppler-broadened transition is given by,

   $$\Delta\nu_{\mathrm{D}} = \sqrt{8\ln 2}\sqrt{\frac{k_{\mathrm{B}}T}{M\mathrm{c}^2}}\nu_0.$$

   where T is the temperature of the atoms, M their mass, and ν_0 the frequency emitted on the transition by a stationary atom.

2. More line broadening: Figure 2.1 shows data from measurements of the homogeneous linewidth of the D_1 ($6p\,^2\mathrm{P}_{1/2} \rightarrow 6\mathrm{s}\,^2\mathrm{S}_{1/2}$) and D_2 ($6p\,^2\mathrm{P}_{3/2} \rightarrow 6\mathrm{s}\,^2\mathrm{S}_{1/2}$) transitions in Cs at 894 and 852 nm respectively.

   (a) Calculate the Doppler width in MHz of these transitions assuming that the temperature of the Cs vapour is 21 °C, and comment on the relative magnitudes of the inhomogeneous and homogeneous linewidths.

   (b) The radiative lifetimes of the D_1 ($6p\,^2\mathrm{P}_{1/2}$ and D_2 ($6p\,^2\mathrm{P}_{3/2}$ levels are 34.75 and 30.41 ns respectively. What is the natural linewidth of the D_1 and D_2 transitions? Is your calculated value consistent with Fig. 2.1?

   (c) Use the data presented to deduce the rate of increase in the homogeneous linewidth in units of $\mathrm{MHz\,Torr}^{-1}$ for each of the two transitions. Explain briefly the cause of this increase in homogenous linewidth with pressure.

   (d) What is the mean collision time at a He pressure of 100 Torr for He-Cs collisions?

   [The molar mass of Cs is 132.9 g.]

3. Steady-state laser oscillation:

   (a) The He-Ne laser operates on several $s \rightarrow p$ transitions in neon, including the $5s \rightarrow 3p$ transition at 632.8 nm. Under the operating conditions of the laser, the fluorescence lifetimes of the upper and lower levels are aproximately 100 ns and 10 ns respectively for this transition, and the Einstein-A coefficient is 10^7 s^{-1}. Taking the upper and lower laser levels to have equal degeneracies, determine whether or not it is possible, in principle, for continuous-wave laser oscillation to be observed on this transition.

   (b) Repeat the calculation for the $3d^{10}4p\ ^2\mathrm{P}_{3/2} \rightarrow 3d^94s^2\ ^2\mathrm{D}_{5/2}$ transition at 510 nm in the copper-vapour laser, given that the Einstein A coefficient is 2×10^6 s^{-1} and the fluorescence lifetime of the lower laser level is approximately 10 μs.

4. Optical gain cross-section:

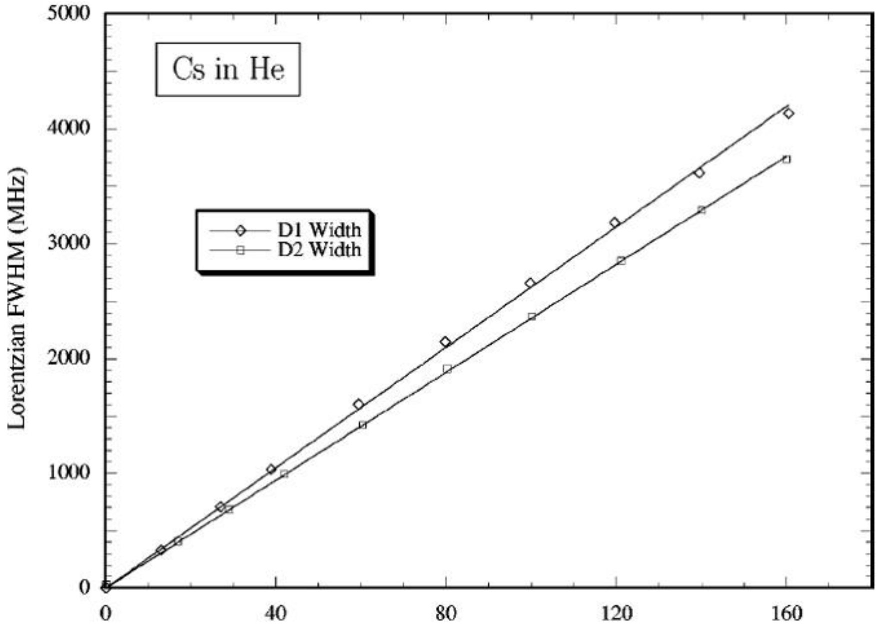

Figure 4.1: Measured full width at half maximum of the homogeneously broadened component of the D_1 and D_2 lines of Cs as a function of the pressure of helium (data from A. Andalkar and R. B. Warrington *Phys. Rev. A* **65** 032708 (2002)).

(a) Show that the optical gain cross-section of a homogeneously broadened laser transition may be written as,

$$\sigma_{21}(\omega - \omega_0) = \frac{\pi^2 c^2}{\omega_0^2} A_{21} g_{\rm H}(\omega - \omega_0), \tag{4.1}$$

where $g_{\rm H}(\omega - \omega_0)$ is the lineshape function of the transition.

(b) Show that for the special case of a purely lifetime broadened transition from an upper level 2 which decays only radiatively to a long-lived lower level 1 the peak optical gain cross-section is given by:

$$\sigma_{21}(0) = \frac{\lambda_0^2}{2\pi}, \tag{4.2}$$

where λ_0 is the vacuum wavelength of the transition.

(c) A laser operates on a transition from an excited electronic energy level of a diatomic molecule in which the two constituent atoms form a molecular bond with each other. This level has a lifetime of 10 ns against radiative decay which is entirely on the laser transition at 250 nm to the unstable ground electronic level, which has a lifetime of 3×10^{-14} s against dissociation into its constituent atoms.

i. Calculate the peak optical gain cross-section of the laser transition, assuming that it is purely lifetime broadened.

ii. What upper level population density would be needed to provide a small signal gain of 0.1cm^{-1}?

iii. Assuming that 10% of the power input leads to formation of molecules in the upper laser level, calculate the minimum power input per unit volume required to sustain the population of the upper level at the value calculated above. Comment briefly on the implications of this result.

5. Gain saturation:

(a) Use a rate equation analysis to show that the gain coefficient of a homogeneously broadened laser transition is modified by the presence of narrow-band radiation of total intensity I to,

$$\alpha_I(\omega - \omega_0) = \frac{\alpha_0(\omega - \omega_0)}{1 + I/I_{\rm s}(\omega_{\rm L} - \omega_0)}, \tag{4.3}$$

where $\omega_{\rm L}$ is the laser frequency. Give an expression for the saturation intensity $I_{\rm s}$.

(b) Explain in physical terms why the saturation intensity depends on the detuning of the intense beam from the centre frequency of the transition.

(c) On the same graph plot the gain coefficient as a function of frequency ω:

i. as measured by a weak probe beam in the absence of any other radiation;

ii. as measured by a weak probe beam in the presence of a narrow-band beam of intensity $I_s(\omega_L - \omega_0)$

iii. as measured by an intense, narrow-band beam of constant intensity $I_s(0)$.

6. A saturated amplifier: A steady-state laser amplifier operates on the homogeneously broadened transition between two levels of equal degeneracy. Population is pumped exclusively into the upper level at a rate of $1.0 \times 10^{18}\ \mathrm{s}^{-1}\,\mathrm{cm}^{-3}$. The lifetimes of the upper and lower levels are 5 ns and 0.1 ns respectively. A collimated beam of radiation enters the 2 m long amplifier with an initial intensity $I(0)$. The gain cross section of the medium is $4 \times 10^{-12}\ \mathrm{cm}^2$ at the 400 nm wavelength of the monochromatic beam. Calculate the intensity of the beam at the exit of the amplifier when:

(a) $I(0) = 0.1\ \mathrm{W\,cm^{-2}}$

(b) $I(0) = 500\ \mathrm{W\,cm^{-2}}$

(c) $I(0) = 50\ \mathrm{W\,cm^{-2}}$

[HINT: In case (c), guess a solution and proceed by iteration]

7. Three- and four-level lasers:

(a) Explain briefly what is meant by the terms **three-level** and **four-level laser**. Discuss why there is a large difference in the threshold power which is required to achieve laser oscillation in these two classes of laser.

(b) A laser cavity is formed by two mirrors of reflectivity 100% and 95%. Calculate the energy absorbed by the active ions which is necessary to achieve pulsed laser oscillation for rods of ruby and Nd:YAG each 50 mm long, 5 mm diameter, and each with an active ion concentration of $4 \times 10^{19}\ \mathrm{cm}^{-3}$. Take the pump bands to be at $20\,000\ \mathrm{cm}^{-1}$ and $12\,000\ \mathrm{cm}^{-1}$ for the ruby and Nd:YAG laser respectively. For the Nd:YAG laser, you may assume that the peak optical gain cross-section is $\sigma_{21}(0) = 6 \times 10^{-19}\ \mathrm{cm}^2$.

(c) How will these values compare with the electrical energy which must be supplied to the laser?