# Phys 673

## Final Examination

### Solutions

$\boxed{1}$ $V(\tau) \propto \Gamma(\tau) = \int \Gamma(\omega') e^{-i\omega'\tau} d\omega' = \Gamma_0 \int_{\omega-\delta\omega/2}^{\omega+\delta\omega/2} e^{-i\omega'\tau} d\omega' = \frac{\Gamma_0}{-i\tau}\left[e^{i(\omega+\delta\omega/2)\tau} - e^{i(\omega-\delta\omega/2)\tau}\right]$

$$= \frac{2\Gamma_0}{\tau} e^{i\omega\tau} \sin \frac{\delta\omega}{2}\tau$$

$\Gamma(\tau) = 0$ if $\frac{\delta\omega}{2}\tau = \pi \Rightarrow \delta\omega = \frac{2\pi}{\tau} = \frac{2\pi c}{l}$

$\boxed{2}$ $|H\rangle \xrightarrow{\lambda/2} \cos 2\theta |H\rangle + \sin 2\theta |V\rangle = \cos 2\theta |1_H 0_V\rangle + \sin 2\theta |0_H 1_V\rangle.$

$$\equiv |\Psi\rangle$$

$$\langle Q_H | \Psi \rangle = \cos 2\theta \, \psi_1(Q) |0_V\rangle + \sin 2\theta \, \psi_0(Q) |1_V\rangle$$

$$= \cos 2\theta \left(\frac{\sqrt{2}\, Q}{\pi^{1/4}} e^{-Q^2/2}\right) |0_V\rangle + \sin 2\theta \left(\frac{1}{\pi^{1/4}} e^{-Q^2/2}\right) |1_V\rangle$$

$\boxed{3}$ a) $I = 2\varepsilon_0 c E_0^2$

$$\Omega = \frac{E_0 d}{\hbar} = \frac{1}{\hbar}\sqrt{\frac{I}{2\varepsilon_0 c}} \sqrt{\frac{3\pi \varepsilon_0 \hbar c^3 \Gamma}{\omega^3}} = \sqrt{\frac{3\pi c^2 I \Gamma}{2\hbar\omega^3}}$$

b) $$\rho_{aa} = \frac{\Omega^2}{\Gamma^2/4 + \Delta^2 + 2\Omega^2}$$

neglect

$$N_{spont} = \Gamma \rho_{aa} A L N = \frac{1}{\Gamma^2/4 + \Delta^2} \frac{3\pi c^2 I \Gamma^2}{2\hbar\omega^3} A L N$$

c) $$\alpha = \frac{N d^2}{\hbar \varepsilon_0} \frac{\omega}{c} \frac{\Gamma/2}{\Gamma^2/4 + \Delta^2} = \frac{N\omega}{\hbar \varepsilon_0 c} \frac{3\pi \varepsilon_0 \hbar c^3}{\omega^3} \frac{\Gamma^2}{2(\Gamma^2/4 + \Delta^2)}$$

$$= \frac{1}{\Gamma^2/4 + \Delta^2} \frac{3\pi N c^2 \Gamma^2}{2\omega^2}$$

$$N_{spont} = \frac{I}{\hbar\omega} A \left(1 - e^{-\alpha L}\right) = \frac{I}{\hbar\omega} A \alpha L = \frac{1}{\Gamma^2/4 + \Delta^2} \frac{3\pi N A L c^2 I \Gamma^2}{2\hbar\omega^3}$$

$\boxed{4}$

$$P(\omega) = N d\, \rho_{ab} = N d\,(i\Gamma/2 - \Delta)\,\frac{\Omega}{\Gamma^2/4 + \Delta^2 + 2\Omega^2}$$

$$= N d\,(i\Gamma/2 - \Delta)\,\Omega\,\frac{1}{(\Gamma^2/4 + \Delta^2)\left(1 + \frac{2\Omega^2}{\Gamma^2/4+\Delta^2}\right)}$$

$$\approx N d\,\Omega\,\frac{-1}{+i\Gamma/2 + \Delta}\left(1 - \frac{2\Omega^2}{\Gamma^2/4+\Delta^2}\right)$$

$$P^{(3)}(\omega) = 3\epsilon_0\,\chi^{(3)}(+\omega,+\omega,-\omega,-\omega)\,E_0^3$$

$$\Rightarrow \chi^{(3)} = \frac{Nd}{3\epsilon_0}\,\frac{+1}{i\Gamma/2+\Delta}\,\frac{2}{\Gamma^2/4+\Delta^2}\left(\frac{\Omega}{E_0}\right)^3 = \frac{2Nd^4}{3\epsilon_0\hbar^3}\,\frac{(-i\Gamma/2+\Delta)}{(\Gamma^2/4+\Delta^2)^2}$$

$$= \frac{2N}{3\epsilon_0\hbar^3}\,\frac{9\pi^2\epsilon_0^2\hbar^2c^6\Gamma^2}{\omega^6}\,\frac{(-i\Gamma/2+\Delta)}{(\Gamma^2/4+\Delta^2)^2} = \frac{6\pi^2\epsilon_0 c^6\Gamma^2 N}{\hbar\omega^6}\,\frac{(-i\Gamma/2+\Delta)}{(\Gamma^2/4+\Delta^2)^2}$$

(See also Boyd sec. 6.3)