University of Calgary
Fall semester 2010

PHYS 673: Quantum and Nonlinear Optics

Final examination

December 16, 2010, 9:00–11:00 (2 hours)

Total points: 100. Open books. No electronic equipment allowed. You must solve all problems in order to receive full credit. Partial credit will be given. Use the booklet provided to write your solutions.

Problem 1 (15 points). A quasi-monochromatic wave of central frequency ω with a rectangular spectral distribution of width $\delta\omega$ enters a Michelson interferometer. The interference fringes first vanish when one of the mirrors is displaced by distance d from the equal path length position. Find $\delta\omega$.

Problem 2 (20 points). A photon of horizontal polarization propagates through a half-wave plate with its optical axis oriented at angle θ to horizontal and then through a polarizing beam splitter that transmits horizontal polarization and reflects vertical. Homodyne detection of the X quadrature is performed in the horizontally polarized transmitted mode and a specific value Q is observed. What is the quantum state of the vertically polarized reflected mode conditioned on this observation?
Note: normalization of the result is not required. There will be no penalty for the ambiguity in the sign of θ associated with the waveplate transformation. The wavefunctions of the first two Fock states are: $\psi_0(X) = (1/\pi^{1/4})e^{-X^2/2}; \psi_1(X) = (\sqrt{2}X/\pi^{1/4})e^{-X^2/2}$.

Problem 3 (35 points). A plane optical wave of frequency ω and intensity I [J/m^2·s] is incident on a cylindrical sample of length L and area A containing a gas of two-level atoms of number density N. The resonant frequency of atoms is ω_0, natural linewidth Γ. Assume that the sample is optically thin and neglect the linear refraction as well as the Doppler broadening. The intensity of light is far below saturation.

a) Find the Rabi frequency.

b) From the steady state density matrix, calculate the average number of photons per second spontaneously emitted by all atoms in the sample.

c) From the absorption coefficient, determine the average number of photons per second absorbed by the whole sample. Verify consistency with (b).

Note: all answers must be given in terms of the quantities listed above and fundamental constants.

EXAM CONTINUES ON THE NEXT PAGE

Problem 4 (30 points). Calculate the third-order nonlinear susceptibility $\chi^{(3)}_{xxxx}(\omega = \omega + \omega - \omega)$ of the atomic gas in Problem 3.

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