Phys 673

Second midterm examination

Solutions

$\boxed{1}$ a) According to Van Cittert - Zernike theorem,

$$V \propto |\Gamma(r_1, r_2)| = \int I(k_\perp)\, e^{i\vec{k}_\perp(\vec{r}_1-\vec{r}_2)}\, d^2\vec{k}_\perp$$

$$\vec{k}_\perp = \frac{\vec{R}}{L} k$$

(where $\vec{R}$ is the radius vector in the plane of the star)

$$\Rightarrow \vec{R} = L\frac{\vec{k}_\perp}{k}$$

$$I(\vec{R}) = I_0 e^{-R^2/w^2}$$

$$I(\vec{k}_\perp) = I_0 e^{-\vec{k}_\perp^{\,2}/k_0^2}, \quad \text{where} \quad k_0 = k\frac{w}{L}$$

Fourier transform of a Gaussian function is a Gaussian function

$$\Rightarrow V = e^{-\frac{1}{4}k_0^2(\vec{r}_1-\vec{r}_2)^2}$$

b) Time delay between two interferometer arms is on the scale of $\frac{|r_1 - r_2|}{c} = \tau$

$\Rightarrow$ need a coherence time of τ

$\Rightarrow$ need filter bandwidth $\approx 1/\tau = \frac{c}{|r_1 - r_2|}$

$\boxed{2}$

a) $e^{-iHt}|1\rangle = \left[1 + \beta t(a^2 - a^{+2})\right]|1\rangle = |1\rangle - \beta t\sqrt{6}\,|3\rangle$

b) $|\alpha\rangle = e^{-|\alpha|^2/2}\left(|0\rangle + \alpha|1\rangle + \frac{\alpha^2}{\sqrt{2}}|2\rangle + \frac{\alpha^3}{\sqrt{6}}|3\rangle\right)$

$|-\alpha\rangle = e^{-|\alpha|^2/2}\left(|0\rangle - \alpha|1\rangle + \frac{\alpha^2}{\sqrt{2}}|2\rangle - \frac{\alpha^3}{\sqrt{6}}|3\rangle\right)$

$N(|\alpha\rangle - |-\alpha\rangle) = N e^{-|\alpha|^2}\left(\alpha|1\rangle + \frac{\alpha^3}{\sqrt{6}}|3\rangle\right)$

$= 2N e^{-|\alpha|^2}\alpha\left(|1\rangle + \frac{\alpha^2}{\sqrt{6}}|3\rangle\right)$

Neglecting second-order contributions,

$N = \frac{1}{2|\alpha|}$

c) $\frac{\alpha^2}{\sqrt{6}} = -\beta t\sqrt{6}$