University of Calgary
Fall semester 2008

PHYS 673: Quantum and Nonlinear Optics

Midterm examination

October 28, 2008

Open books. Attempt all problems.
Full credit = 100 points. Partial and extra credit will be given.

Problem 1. Three plane waves with frequencies ω_1, ω_2, ω_3 and intensities I_1, I_2, I_3 propagate through an isotropic nonlinear medium in positive x, y and z directions, respectively. The electric fields in waves 1 and 2 are polarized, respectively, along the y and z axes. Wave 3 is circularly polarized. One of the components of the third-order nonlinear susceptibility $\chi^{(3)}_{xxxx} \equiv \chi$ and the index of refraction $n(\omega)$ are known. Under these circumstances, the sum-frequency nonlinear polarization is of the form

$$\vec{P} = \vec{P}_0 e^{i\vec{k}\vec{r} - i(\omega_1+\omega_2+\omega_3)t} + \text{c.c.}$$

a) Write the three components of the vector $\vec{k}$ (10 pts).

b) Find the direction of P_0 (10 pts).

c) Find the magnitude of P_0 (20 pts).

d) What is the condition on $n(\omega)$ that would ensure phase matching for the sum frequency emission (10 pts)?

Problem 2. Consider the evolution under the Hamiltonian

$$\hat{H} = \alpha[\hat{a}^2 + (\hat{a}^\dagger)^2]$$

with a real, positive α.

a) Find the differential equations for the annihilation, creation, position, and momentum observables in the Heisenberg picture (20 pts).

b) Map the evolution in the phase space. Sketch the Wigner function of the state into which the vacuum state will transform into under this Hamiltonian (30 pts).

c) By calculating the variance of the appropriate quadrature, show this state to be squeezed (20 pts).

Problem 3. Light from an incandescent bulb is coupled into a single-mode fiber with the index of refraction n and transmitted through a narrowband filter with central wavelength λ and FWHM $\delta\omega$. The light is then split and recombined to form an interferometer (Fig.). One of the path lengths is slowly varied to observe interference fringes.

a) Write the visibility of the fringes as a function of the path-length difference L (20 pts).

b) Do we need to apply lowpass filtering (time averaging) to the detector photocurrent in order to obtain a reliable measurement, and if so, what should be the cutoff frequency (averaging time) and why (10 pts)?

Assume the filter to be Gaussian.