Phys 673

Midterm examination

Solutions

$\boxed{1}$ a) $\vec{P} = \varepsilon_0 \overleftrightarrow{\chi}^{(3)} \vec{E}_1 \vec{E}_2 \vec{E}_3$

$$\vec{P}_{SFG} = \varepsilon_0 \overleftrightarrow{\chi}^{(3)}_{eff} \vec{E}_{10} \vec{E}_{20} \vec{E}_{30} e^{i\vec{k}_1\vec{r} - i\omega_1 t} e^{i\vec{k}_2\vec{r} - i\omega_2 t} e^{i\vec{k}_3\vec{r} - i\omega_3 t} + c.c.$$

$$\vec{k}_1 = \frac{n(\omega_1)\omega_1}{c}\hat{x}$$

$$\vec{k}_2 = \frac{n(\omega_2)\omega_2}{c}\hat{y}$$

$$\vec{k}_3 = \frac{n(\omega_3)\omega_3}{c}\hat{z}$$

$$\vec{k} = \vec{k}_1 + \vec{k}_2 + \vec{k}_3 \Rightarrow k_x = k_1 \quad k_y = k_2 \quad k_z = k_3$$

b) The only nonvanishing components of $\overleftrightarrow{\chi}^{(3)}$ are $\chi^{(3)}_{ijij}$, $\chi^{(3)}_{iijj}$, $\chi^{(3)}_{ijji}$ and $\chi^{(3)}_{iiii}$, which is the sum of the above three. Given polarizations of waves 1 and 2, the 2nd index is y, 3rd index is z. The only relevant component of $\overleftrightarrow{\chi}^{(3)}$ is thus $\chi^{(3)}_{zyzy}$ ($\chi^{(3)}_{yyzz}$ is irrelevant because wave 3 must be transverse). Hence $\vec{P}_0$ is along $\hat{z}$

c) $$P_0 = \varepsilon_0 \chi^{(3)}_{zyzy} E_{10} E_{20} \frac{E_{30}}{\sqrt{2}}$$

(factor $\frac{1}{\sqrt{2}}$ because only the y-component of wave 3 matters)

Given $I = 2n\varepsilon_0 c E_0^2$, $E_{i0} = \sqrt{\frac{I_i}{2n(\omega_i)\varepsilon_0 c}}$

$$P_0 = \frac{1}{4\sqrt{\varepsilon_0}\, c^{3/2}} \frac{\sqrt{I_1 I_2 I_3}}{\sqrt{n(\omega_1)n(\omega_2)n(\omega_3)}}$$

d) Must have $k = \frac{n(\omega_1+\omega_2+\omega_3)(\omega_1+\omega_2+\omega_3)}{c}$

On the other hand, $k = \frac{1}{c}\sqrt{n^2(\omega_1)\omega_1^2 + n^2(\omega_2)\omega_2^2 + n^2(\omega_3)\omega_3^2}$

Thus $$n(\omega_1+\omega_2+\omega_3) = \frac{\sqrt{n^2(\omega_1)\omega_1^2 + n^2(\omega_2)\omega_2^2 + n^2(\omega_3)\omega_3^2}}{\omega_1+\omega_2+\omega_3}$$

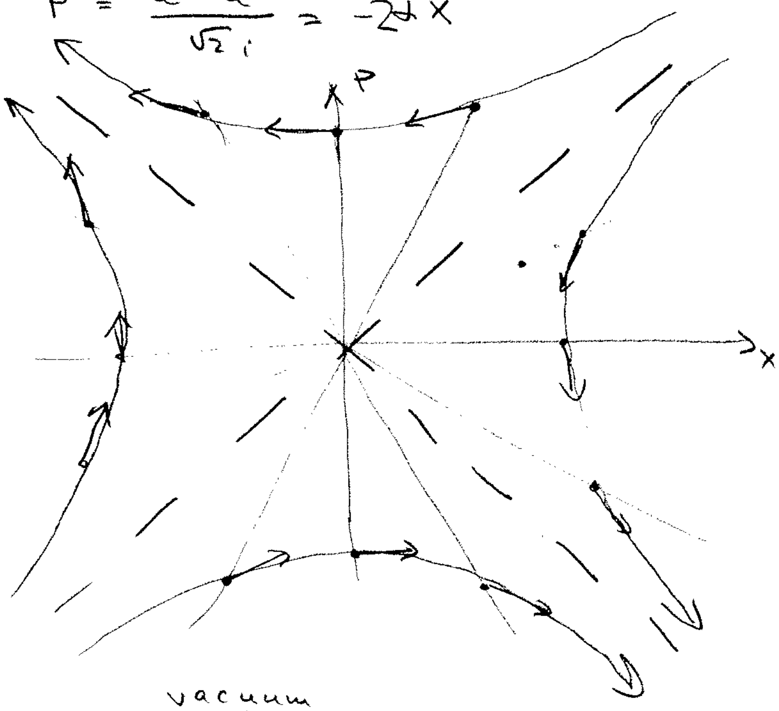
$\boxed{2}$ a) $\dot{a} = i\,[H\ a] = -2i\alpha a^+$

$$\dot{a}^+ = 2i\,\alpha a$$

$$\dot{X} = \frac{\dot{a} + \dot{a}^+}{\sqrt{2}} = -2\alpha P$$

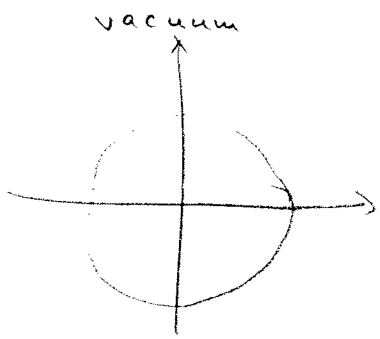
$$\dot{P} = \frac{\dot{a} - \dot{a}^+}{\sqrt{2}\, i} = -2\alpha X$$

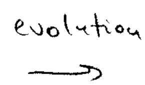
b)

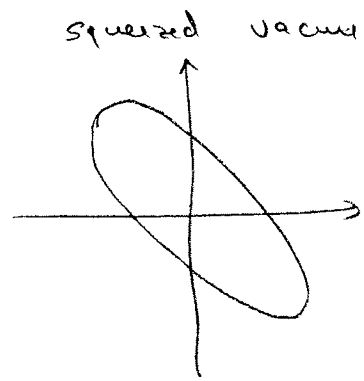


First calculate the "velocity" vector for some points in phase space, then combine them into trajectories

We obtain squeezing of $(X+P)$, stretching of $(X-P)$







c) $$X_+ = \frac{X+P}{\sqrt{2}}$$

$$\dot{X}_+ = \frac{\dot{X}+\dot{P}}{\sqrt{2}} = -2\alpha X_+ \Rightarrow X_+ = X_{+0}\, e^{-2\alpha t}$$

$$\langle X_+^2 \rangle = \frac{1}{2} e^{-4\alpha t}$$

$$X_- = \frac{X-P}{\sqrt{2}}$$

$$\dot{X}_- = \frac{\dot{X}-\dot{P}}{\sqrt{2}} = 2\alpha X_- \Rightarrow X_- = X_{-0}\, e^{2\alpha t}$$

$$\langle X_-^2 \rangle = \frac{1}{2} e^{4\alpha t}$$

3 a) As discussed in class,

$$V = \left| \Gamma \left(\tau = \frac{nL}{c} \right) \right|$$

According to the Wiener-Khinchine theorem,

$\Gamma(\tau)$ is the Fourier transform of the filter transmission function. $T(\omega) = e^{-\frac{(\omega-\omega_0)^2}{(\delta\omega/2)^2 \ln 2}}$

$$\Gamma(\tau) = FT\left[e^{-\frac{8}{\delta\omega^2 \ln 2} \frac{(\omega-\omega_0)^2}{2}} \right] = e^{i\omega_0\tau} e^{-\frac{\delta\omega^2 \ln 2}{8} \frac{\tau^2}{2}}$$

$$V = e^{-\frac{\delta\omega^2 \ln 2}{16} \left(\frac{nL}{c}\right)^2}$$

b) Must integrate over the coherence time, i.e. $\sim \frac{1}{\delta\omega}$