**University of Calgary**
**Fall semester 2008**

## PHYS 673: Quantum and Nonlinear Optics

# Homework assignment 3

Due October 21, 2008. Computers are not allowed except in Problem 3.2

Problem 3.1. Verify for the photon statistics of a coherent state $|\alpha\rangle$ that $\langle n\rangle = \left\langle \Delta n^2 \right\rangle = |\alpha|^2$.

Problem 3.2. Use Mathematica or Matlab to calculate and plot the wave functions in the position basis (wherever they exist) and Wigner functions of the following states:

a) vacuum state;

b) coherent state with $\alpha = (1+i)/\sqrt{2}$;

c) the single-photon state;

d) the ten-photon state;

e) $(|0\rangle + |1\rangle)/\sqrt{2}$;

f) $(|0\rangle + i\,|1\rangle)/\sqrt{2}$;

g) state with the density matrix $(|0\rangle\langle 0| + |1\rangle\langle 1|)/2$;

h) $(|0\rangle + |2\rangle)/\sqrt{2}$;

i) pure squeezed state with $\left\langle \Delta X^2 \right\rangle = 1/4$;

j) Schrödinger cat states $|\alpha\rangle \pm |-\alpha\rangle$ with $\alpha = 3$ and $\alpha = 1$ (please also calculate the norm for these states);

k) state with the density matrix $(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle -\alpha|)/2$ with $\alpha = 3$.

Verify normalization of Wigner functions. For each of these states, integrate the Wigner functions to obtain $\mathrm{pr}(X)$ and $\mathrm{pr}(P)$. Plot these functions. Verify that $\mathrm{pr}(X) = |\psi(X)|^2$ (wherever $\psi(X)$ exists). Calculate $\left\langle \Delta X^2 \right\rangle$, $\left\langle \Delta P^2 \right\rangle$.

Problem 3.3. Find the range of $\beta$'s for which the state $|\psi\rangle = (|0\rangle + \beta\,|2\rangle)/\sqrt{1+\beta^2}$ is position squeezed. **Hint:** there is no need to calculate the wave function.

Problem 3.4. Consider the evolution under the squeezing Hamiltonian

$$\hat{H} = i\alpha[\hat{a}^2 - (\hat{a}^\dagger)^2]$$

with a real, positive $\alpha$ in the Schrödinger picture. Verify that the wavefunction

$$\psi(X) = \frac{\sqrt{s}}{\pi^{1/4}} e^{-s^2 X^2/2}$$

with $s = -2\alpha t$ is a solution to the Schrödinger equation.

Problem 3.5. Consider the evolution of a two-mode state under the Hamiltonian

$$\hat{H} = 2i\alpha(\hat{a}_1 \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_2^\dagger),$$

with a real, positive $\alpha$ in the Heisenberg picture. Find the differential equations for the four quadrature operators as well as $(\hat{X}_1 \pm \hat{X}_2)/\sqrt{2}$ and $(\hat{P}_1 \pm \hat{P}_2)/\sqrt{2}$. Find the variances of these observables as functions of time is the initial state is vacuum. Discuss relevance of the generated *two-mode squeezed state* to the original Einstein-Podolsky-Rosen state.