University of Calgary
Fall semester 2008

PHYS 673: Quantum and Nonlinear Optics

Homework assignment 6

Due December 4, 2008. Computers are allowed.

Problem 6.1. Read the paper "Demonstration of an all-optical quantum controlled-NOT gate" by J. L. O'Brien *et al.*, Nature **426**, 264 (2003). Verify functionality of the scheme displayed in Fig. 1(a) using the beam splitter operator in the Fock basis. **Note:** it may be necessary to include certain phase shifts into the beam splitter matrices.

Problem 6.2. *Electromagnetically-induced transparency (EIT)* is an effect observed in atoms with a Λ-shaped energy level structure. There are two ground levels $|b\rangle$, $|c\rangle$ and one excited level $|a\rangle$. Spontaneous emission rates from $|a\rangle$ into $|b\rangle$ and $|c\rangle$ are Γ_b and Γ_c, respectively. There are two electromagnetic fields: the *control* field with Rabi frequency Ω_c coupling $|c\rangle$ with $|a\rangle$ with detuning Δ_c and the *signal* field with Rabi frequency Ω_b coupling $|b\rangle$ with $|a\rangle$ with detuning Δ_b. There is no population exchange between levels $|b\rangle$ and $|c\rangle$, but there is decoherence manifesting itself as decay of the matrix element ρ_{bc} with rate $\gamma \ll \Gamma_b, \Gamma_c$.

a) Write the interaction picture Hamiltonian assuming level $|b\rangle$ to be of zero energy.

b) Write a full set of master equations for the nine density matrix elements.

c) Hereafter assume that the signal field is very weak: its Rabi frequency is much smaller than all other Rabi frequencies and decay rates involved. In which state will most of the atomic population be collected? Provide a qualitative argument.

d) Because populations of other levels are small, the evolution problem can be solved in the pure state approximation. Write the Schrödinger equation for the amplitudes ψ_a, ψ_b and ψ_c.

e) Find the steady state amplitudes.

f) Assume $\Gamma_b = \Gamma_c \equiv \Gamma$ and plot the real and imaginary parts of the atomic medium's susceptibility with respect to the signal field as a function of Δ_b for the following parameter sets:

- $\Delta_c = 0, \Omega_c = 0, \gamma = 0$;

- $\Delta_c = 0, \Omega_c = \Gamma/2, \gamma = 0$;
- $\Delta_c = 0, \Omega_c = \Gamma/2, \gamma = \Gamma/8$;
- $\Delta_c = 3\Gamma, \Omega_c = \Gamma/2, \gamma = 0$;
- $\Delta_c = 0, \Omega_c = 8\Gamma, \gamma = 0$.

For each case, provide a brief qualitative discussion of the behavior observed.

g) Obtain an approximate analytic expression for the FWHM of the EIT window for the case $\Delta_c = 0, \Omega_c \ll \Gamma, \gamma = 0$.

h) Obtain an approximate analytic expression for the ratio of the absorption coefficients in the middle and outside the EIT window for the case $\Delta_c = 0, \Omega_c \ll \Gamma, \gamma \ll \Omega_c$.