University of Calgary
Fall semester 2008

PHYS 673: Quantum and Nonlinear Optics

Homework assignment 1

Due September 23, 2008

Problem 1.1. In class, we used the classical theory of dispersion to derive the susceptibility of a gas of linear oscillators. Verify the Kramers-Kronig relations for the linear and imaginary parts of this susceptibility. Near-resonance, narrow-resonance approximations can be used.

Problem 1.2. Use the classical theory of dispersion to calculate the index of absorption of a gas at wavelength $\lambda = 800$ nm with the resonance width $\gamma/2\pi = 6$ MHz and number density 10^{10} cm^{-3}.

a) Assume that the oscillators do not move.

b) Take into account Doppler broadening. Assume room temperature. The mass of each oscillator is equal to that of a rubidium atom. **Hint:** because the natural oscillator's linewidth is much narrower than the Doppler width, the natural resonance of each atom can be assumed to be the delta-function.

Problem 1.3. Consider a slab of some substance of length L. The absorption index of the substance is equal to α at all frequencies except a narrow line of width γ at frequency ω_0. The transparency window is of Lorentzian shape.

a) Write the imaginary susceptibility χ'' and use the Kramers-Kronig relations to calculate the real susceptibility χ'.

b) Calculate the group velocity v_{gr} on resonance. Show it to be subluminal.

c) A laser pulse of frequency ω_0 and temporal width $\tau \approx 1/\gamma$ (so its spectrum just fits into the transparency window) propagates through the medium. Find the delay compared to propagation through vacuum. Express it in the units of τ.

Problem 1.4. Show that in an isotropic non-chiral medium, the only nonvanishing elements of the third order nonlinear susceptibility are $\chi^{(3)}_{iijj}$, $\chi^{(3)}_{ijij}$, and $\chi^{(3)}_{ijji}$ (where i, j are x, y, z). Show that each of these elements is the same for all i and j as long as $i \neq j$ (i.e., for example, $\chi^{(3)}_{xxyy} = \chi^{(3)}_{yyzz}$). Show that

$$\chi^{(3)}_{xxxx} = \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx}.$$

Problem 1.5. You need to generate second harmonic of continuous-wave laser light at wavelength $\lambda = 1064$ nm in a beta barium borate (BBO) crystal of length $L = 5$ mm cut for optimal phase matching. Use the data from Wikipedia to answer the following questions.

a) At which angle to the optical axis should the beam propagate to achieve phase matching?

b) Which wave (fundamental or second harmonic) is ordinary, which extraordinary?

c) Find the FWHM of the phase matching band (in units of wavelength).

d) Find the acceptance angle.

e) Find the walk-off angle for both waves.

f) The power of the fundamental beam is $P = 100$ mW. Estimate the power of generated second harmonic assuming optimal beam geometry.

Problem 1.6. You need to generate second harmonic of continuous-wave laser light at wavelength $\lambda = 1064$ nm in a periodically poled potassium titanyl phosphate (PPKTP) crystal of length $L = 5$ mm. Light propagates along the x axis and is polarized along the z axis. Find the necessary information about the crystal on the Internet and answer the following questions.

a) What is the poling period required for phase matching?

b) Find the FWHM of the phase matching band.

c) The power of the fundamental beam is $P = 100$ mW. Find the power of generated second harmonic assuming optimal beam geometry.