University of Calgary
Fall semester 2014

PHYS 615: Advanced Quantum Mechanics I

Midterm examination

October 31, 2013, 16:00

Open books. No electronic equipment allowed.
Full credit = 100 points. Attempt all problems. Partial credit will be given.

Problem 1 (10). Find the commutator $[(\hat{\sigma}_x + \hat{\sigma}_y)^2, \hat{\sigma}_z]$.
Problem 2 (20). The density matrix of a photon's state in the canonical basis is

$$\hat{\rho} = \begin{pmatrix} 1/2 & i/6 \\ -i/6 & 1/2 \end{pmatrix}.$$

Present this state as a statistical mixture of orthogonal pure states.
Problem 3 (25). An ensemble of spin-1/2 particles initially in state $|\uparrow\rangle$ undergoes decoherence due to collisions with a buffer gas. Each collision results in complete decoherence of the particle that experienced it. The decoherence preferred basis is $\{|\pm\rangle\} = \{(|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}\}$. The probability of collision per particle per unit time is p. Write the density matrix as a function of time

a) in the decoherence preferred basis;

b) in the canonical basis.

Problem 4 (25). Suppose a photon's polarization is measured in the circular basis using a device consisting of a quarter-wave plate, a PBS and a pair of single-photon detectors with efficiency η.

a) Write the POVM of this device.

b) Find the probability of each outcome for input state $\alpha |H\rangle + \beta |V\rangle$.

Problem 5 (20). Alice and Bob share two photons in state $|\Psi\rangle = (|HV\rangle + |VH\rangle + 2|VV\rangle)/\sqrt{6}$. Alice measures the state in the diagonal basis.

a) What state will be prepared at Bob's station in each case?

b) What is the probability of each outcome?

c) Write the density matrix of the state prepared at Bob's station if Bob does not know Alice's result in the canonical basis.