University of Calgary
Fall semester 2014

PHYS 615: Advanced Quantum Mechanics I

Homework assignment 2

Due Monday October 6, 2014

Problem 2.1. Alice and Bob share state $|\Psi\rangle = (|HH\rangle + i|VH\rangle - 3|VV\rangle)/\sqrt{11}$.

a) Find the expectation value and uncertainty of operator $\hat{\sigma}_x \otimes \hat{\sigma}_z$ in this state.

b) Find partial inner products $\langle R_{\text{Alice}}|\Psi\rangle$ and $\langle\Psi|R_{\text{Bob}}\rangle$, where $|R\rangle$ is the right circular polarization state.

c) Alice measures state $|\Psi\rangle$ in the canonical basis. What is the probability of each outcome and what state will be prepared at Bob's station in each case?

d) Suppose Bob does not know Alice's result. Based on part (c), give a verbal description of the state of the photon at Bob's station.

e) Repeat parts (c) and (d) for Alice's measurement in the $|\pm 45^\circ\rangle$ basis.

f) State $|\Psi\rangle$ is used in the teleportation protocol as the entangled resource instead of the singlet Bell state. Find the probability for each of Alice's Bell basis measurement results and the corresponding state emerging in Bob's channel, if the state to be teleported is $\chi = \alpha|H\rangle + \beta|V\rangle$.

Problem 2.2. Reproduce the GHZ argument for

$$\left|\Psi'_{GHZ}\right\rangle = \frac{1}{2}(|HHH\rangle + |HVV\rangle + |VVH\rangle + |VHV\rangle) \tag{1}$$

and operators

$$\begin{array}{c}
\hat{\sigma}_z \otimes \hat{\sigma}_y \otimes \hat{\sigma}_y \\
\hat{\sigma}_y \otimes \hat{\sigma}_z \otimes \hat{\sigma}_y \\
\hat{\sigma}_y \otimes \hat{\sigma}_y \otimes \hat{\sigma}_z \\
\hline
\hat{\sigma}_z \otimes \hat{\sigma}_z \otimes \hat{\sigma}_z
\end{array}$$

Problem 2.3. A quantum communication line between Alice and Bob consists of $k = 10$ quantum repeater links (Fig. 1). Each link consists of

- two perfect sources of photon pairs in a Bell state with an emission rate of $f = 10^6$ pairs per second;
- two perfect quantum optical memory cells located near the sources;
- two fiber links of length $L = 25$ km each with loss coefficient $\beta = 0.2$ dB/km;
- a perfect Bell-state analyzer.

The Bell-state analyzer in each link "clicks" (produces an output) if the photons from both sources reach it. In this event, the counterparts of these photons become entangled thanks to the entanglement swapping effect. This entangled pair is stored in the two memory cells. Once the memory cells of all the links is filled, a Bell-state measurement is performed on each pair of neighboring cells. In this way, an entangled state is established between Alice's and Bob's locations.

a) For a single link, find the probability to obtain entanglement in its memory cells after a single attempt.

b) For a single link, find the probability to obtain entanglement in its memory cells after N attempts.

c) Find the probability to obtain entanglement in all k links after N simultaneous attempts in each link.

d) Find the time required to obtain entanglement between Alice's and Bob's memory cells with a probability of at least 1/2.

e) Instead of using a quantum repeater, Alice sends photons directly to Bob via a fiber line of length $2kL = 500$ km using a photon source with an emission rate of $f = 10^6$ photons per second. Find t such that at least one of the photons sent by Alice during a time period t reaches Bob with a probability of 1/2 .

All answers in this problem must be presented in both symbolic and numeric forms.
Hint: $(1 + \epsilon)^M \approx e^{\epsilon M}$ for small ϵ, large M.

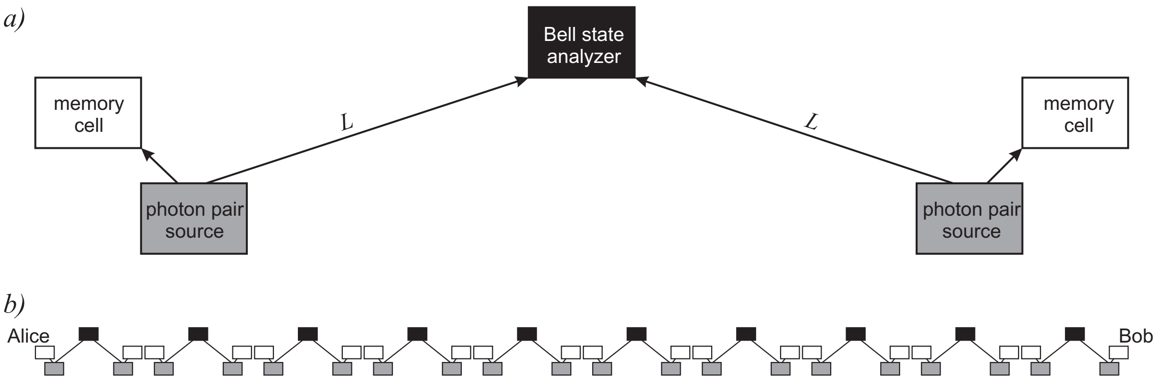

Figure 1: Quantum repeater. a) Individual link; b) entire line.