University of Calgary
Fall semester 2014

PHYS 615: Advanced Quantum Mechanics I

Homework assignment 3

Due Monday October 20, 2014

Problem 3.1. Consider the ensemble of photons that is

- in state $|\psi_1\rangle = (3|H\rangle - 4|V\rangle)/5$ with probability $p_1 = 1/2$;
- in state $|\psi_2\rangle = (12|H\rangle - 5i|V\rangle)/13$ with probability $p_2 = 1/4$;
- in state $|\psi_3\rangle = |-45^\circ\rangle$ with probability $p_3 = 1/4$.

a) Find the density operator.

b) This ensemble is measured in the circular basis. Find the probabilities of each result using the verbal description above and using the density matrix formalism. Verify consistency.

c) Present the ensemble as a mixture of two orthogonal states.

The answers should be in a numerical form, up to the third decimal point.

Problem 3.2. A polarization measurement device consisting of a PBS and two perfect photon detectors contains a "gremlin" who with probability 1/2 inserts a half-wave plate with its optical axis oriented at $\pi/4$ before the PBS. Find the POVM of that detector.

Problem 3.3. A quantum process $\mathbf{E}$ on a polarization qubit has been subjected to a quantum process tomography experiment. It has revealed the following transformations of the probe states:

$$|H\rangle \to 1/4\,|H\rangle\langle H| + 3/4\,|V\rangle\langle V|\,;$$
$$|V\rangle \to 3/4\,|H\rangle\langle H| + 1/4\,|V\rangle\langle V|\,;$$
$$|+\rangle \to |+\rangle\langle +|\,;$$
$$|R\rangle \to 1/2\,|H\rangle\langle H| + 1/2\,|V\rangle\langle V| + i/4\,|H\rangle\langle V| - i/4\,|V\rangle\langle H|\,.$$

Find the process tensor $\mathbf{E}_{lk}^{nm}$ such that

$$[\mathbf{E}(\hat{\rho})]_{lk} = \sum_{nm} \mathbf{E}_{lk}^{nm} \rho_{nm}.$$

For convenience, please present your answer in the form of sixteen equation such as $\mathbf{E}_{HH}^{HH} = \ldots$, etc.

How will the process transform states $|-\rangle$, $|L\rangle$, $p\,|H\rangle\langle H| + (1-p)\,|-\rangle\langle -|$? What is the decoherence preferred basis?

Problem 3.4. Alice and Bob share state $|\Psi\rangle = (|HH\rangle + i\,|VH\rangle - 3\,|VV\rangle)/\sqrt{11}$.

a) Determine Bob's reduced density matrix using the partial trace and check for consistency with the previous homework problem 2.1(d).

b) What is the probability of each result and the state prepared with Bob if Alice measures $|\Psi\rangle$ using the detector from Problem 3.2?

Problem 3.5. Consider an ensemble of electrons in initial state $|\psi(t=0)\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. The electrons are in vacuum and subjected to a magnetic field $\vec{B}$ pointing in the z direction, so the Hamiltonian is $\hat{H} = \hbar\mu_B \hat{\vec{\sigma}} \cdot \vec{B}$. Suppose the ensemble experiences decoherence so each electron, with probability $\gamma \ll \mu_B B/\hbar$ per unit time, collides with one of the residual atoms that are present due to imperfect vacuum. The decoherence-preferred basis is the energy eigenbasis.

a) Write the differential equation for the evolution of the density matrix, taking decoherence into account.

b) Solve that equation and find $\hat{\rho}(t)$ in the $m_s = \pm 1/2$ basis.

c) Check your solution by solving the Schrödinger equation for a pure state in the absence of decoherence.

d) Find the expectation value of the x-component of the spin and plot it as a function of time.

Hint: Express $\hat{\rho}(t)$ as a linear combination of Pauli matrices and use the commutation relations for Pauli matrices to obtain the system of differential equations for the evolution.