University of Calgary
Fall semester 2014

PHYS 615: Advanced Quantum Mechanics I

Homework assignment 6

Due Monday December 8, 2014
Computers are not allowed except in Problem 1.

Problem 6.1. Use a mathematical software package to calculate the Wigner function of the states below. Also the probability densities $\mathrm{pr}(X)$ and $\tilde{\mathrm{pr}}(P)$ for the position and momentum from the states' wavefunctions. Verify that these probability densities are obtained when the Wigner function is integrated over the other quadrature. Plot both probability densities and the Wigner function. If you have no 3D graphics software, plot relevant cross-sections of the Wigner function to show their shapes.

a) vacuum state;

b) coherent state with $\alpha = 2$;

c) the single-photon state;

d) the four-photon state;

e) $(|0\rangle + |1\rangle)/\sqrt{2}$;

f) $(|0\rangle + i\,|1\rangle)/\sqrt{2}$;

g) state with the density matrix $(|0\rangle\langle 0| + |1\rangle\langle 1|)/2$;

h) squeezed vacuum state $\psi_r(X) = \pi^{-1/4}\sqrt{r}e^{-x^2/2r^2}$; plot for $r = 2$;

i) Schrödinger cat state $|\alpha\rangle - |-\alpha\rangle$ with real $\alpha = 3$ (neglect normalization); plot for $\alpha = 3$.

j) state with the density matrix $(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)/2$; plot for $\alpha = 3$.

Problem 6.2. The optical field in a certain mode is initially in the vacuum state: $|\psi(0)\rangle = |0\rangle$. The evolution in Heisenberg picture occurs according to

$$\begin{aligned}\hat{X}(t) &= \hat{X}(0) + \epsilon\hat{P}(0)t;\\ \hat{P}(t) &= \hat{P}(0),\end{aligned}$$

with a real $\epsilon > 0$.

a) Find a Hamiltonian that results in such evolution.

b) Plot the Wigner function of the state for $\epsilon t = 2$.

c) Show that by applying an optical phase shift by a certain angle $\theta(t)$, state $|\psi(t)\rangle$ can be transformed into a pure position squeezed state. Find θ, as well as the variances of the position and momentum quadratures in the latter state.
Hint: Applying the phase shift rotates the Wigner function by θ.

Problem 6.3. A weak coherent state $|\alpha\rangle$ in input mode 1 of the beam splitter is overlapped with the single-photon state in mode 2 (the notation is the same as in the lecture notes). In the beam splitter's output mode 1, a perfect single-photon detector is placed. Find the state in output mode 2 to the first order in α and the amplitude reflectivity r assuming $\alpha \sim r \ll 1$. Consider two cases:

a) an ideal single photon source;

b) an inefficient single-photon source that produces the state $\eta\,|1\rangle\langle 1| + (1-\eta)\,|0\rangle\langle 0|$, where η is that source's efficiency.

Problem 6.4. Consider a squeezed vacuum state with squeezing parameter ζ that has passed through an attenuator with intensity transmissivity η.

a) Using the beam splitter model of absorption in the Heisenberg picture, find the position and momentum uncertainties of the transmitted state.

b) Express the initial state in the Fock basis to the first order of ζ. Then use the beam splitter model of absorption in the Schrödinger picture to find the density matrix of the transmitted state in the Fock basis. Determine the position and momentum uncertainties from that density matrix and verify consistency with part (a).