University of Calgary
Fall semester 2014

PHYS 615: Advanced Quantum Mechanics I

Homework assignment 1

Due Monday September 22, 2014
Using symbolic computation software strongly recommended

Problem 1.1. A measurement apparatus consists of a half-waveplate with its extraordinary axis oriented at angle 45° to horizontal, followed by a quarter-wave plate with its extraordinary axis oriented at angle 30° to horizontal, followed by a polarizing beam splitter which sends horizontally and vertically polarized photons to different detectors labeled 1 and 2, respectively.

a) Write the evolution operator matrices for each of the waveplates and for their combination in the canonical basis.

b) Write the measurement eigenstates in the canonical basis.

c) Plot the trajectory of the tip of the electric field vector corresponding to these states.

d) Values 1 and -1 are associated with the "clicks" of detectors 1 and 2, respectively. Write the matrix of the corresponding observable.

Hint: the quarter-waveplate with its extraordinary axis oriented horizontally is associated with operator $|H\rangle\langle H| + i\,|V\rangle\langle V|$.

Problem 1.2. Consider two operators:

$$\hat{A} = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}, \quad \hat{B} = \hat{\sigma}_z.$$

a) Show that these operators are Hermitian (i.e. they can be interpreted as physical observables).

b) Are these operators unitary?

c) Find their eigenvalues and eigenstates.

d) Find their commutator.

e) The observable $\hat{A}$ is measured in the state $|\psi_0\rangle = (3i\,|H\rangle + 4\,|V\rangle)/5$. What results can be obtained and with which probabilities?

f) Find the expectation values and variances of the measurements of $\hat{A}$ and $\hat{B}$ in state $|\psi\rangle$.

g) Verify that the uncertainty principle holds for the measurements in part (f).

Problem 1.3. Consider an operator $\hat{A}$ that performs the following transformation.

$$|R\rangle \rightarrow \frac{4\,|V\rangle + 3i\,|H\rangle}{5}; \tag{1}$$

$$|-\rangle \rightarrow \frac{2-i}{\sqrt{5}}\,|+\rangle\,. \tag{2}$$

a) Write the matrix of $\hat{A}$ in the canonical basis.

b) Determine how $\hat{A}$ acts upon both circular polarization states.

c) Using the previous result, find the matrix of $\hat{A}$ in the circular polarization basis;

d) Find the matrix of $\hat{A}$ in the canonical basis from its matrix in the circular basis using the method of "inserting $\hat{\mathbf{1}}$". Is your result consistent with that of part (b)?

e) Find the traces of the matrices of $\hat{A}$ in the canonical and circular bases. Are they identical?

f) Express $\hat{A}$ in the Dirac notation in terms of outer products of states $|H\rangle$ and $|V\rangle$;

g) Is $\hat{A}$ Hermitian? If not, what is its adjoint? Is $\hat{A}$ unitary?

<u>Problem 1.4.</u> A photon enters a slab of birefringent material of thickness L with its extraordinary axis oriented at angle φ to horizontal. The ordinary and extraordinary refractive indices are n_o and n_e, respectively.

a) Write the effective Hamiltonian that governs the evolution of the photon in this material.

b) Find the evolution operator.

c) Using this result, find the state of the photon after propagating through the slab if the initial state is (i) horizontally polarized and (ii) right circularly polarized.

d) Repeat part (c) by solving the Schrödinger equation with the same initial states.

e) At which length does the slab act as a half-wave plate? Verify that your answer is consistent with those of parts (c) and (d).