**University of Calgary**
**Fall semester 2014**

## PHYS 615: Advanced Quantum Mechanics I

# Homework assignment 4

Due Monday November 10, 2014

Problem 4.1. Find the generators of the SO(3) rotation group and their commutation relations. Compare these commutation relations with those for the angular momentum operators.

Problem 4.2 [Sakurai #8]. Consider a sequence of Euler rotations represented by

$$\begin{aligned}\mathcal{D}^{1/2}(\alpha,\beta,\gamma) &= \exp\left(\frac{-i\hat{\sigma}_3\alpha}{2}\right)\exp\left(\frac{-i\hat{\sigma}_2\beta}{2}\right)\exp\left(\frac{-i\hat{\sigma}_3\gamma}{2}\right)\\ &= \begin{pmatrix} e^{-i(\alpha+\gamma)/2}\cos\frac{\beta}{2} & -e^{-i(\alpha-\gamma)/2}\sin\frac{\beta}{2} \\ e^{i(\alpha-\gamma)/2}\sin\frac{\beta}{2} & e^{i(\alpha+\gamma)/2}\cos\frac{\beta}{2}\end{pmatrix}\end{aligned}$$

Because of the group properties of rotations, we expect that this sequence of operations is equivalent to a *single* rotation about some axis by an angle $\theta$. Find $\theta$.

Problem 4.3 [Sakurai #12]. An angular momentum eigenstate $|j, m = m_{\max} = j\rangle$ is rotated by an infinitesimal angle $\varepsilon$ about the $y$-axis. Without using the explicit form of the $d^{(j)}_{m'm}$ function, obtain an expression for the probability for the new rotated state to be found in the original state up to terms of order $\varepsilon^2$.

Problem 4.4 Compute all the Clebsch-Gordan coefficients relevant for the coupling of $j_1 = 1$ and $j_2 = 1/2$. Using this result, evaluate $(\hat{J}_1 \cdot \hat{J}_2)\,|j_1, j_2, j, m\rangle$ for all possible values of $j$ and $m$.