**University of Calgary**
**Fall semester 2013**

**PHYS 615: Advanced Quantum Mechanics I**

# Midterm examination

October 25, 2013, 11:00 am

Open books. No electronic equipment allowed.
Full credit = 100 points. Attempt all problems. Partial credit will be given.

Problem 1 (50 pts). For the *photon-added coherent state* $\hat{a}^\dagger |\alpha\rangle$:

   a) Find its photon statistics $\mathrm{pr}(n)$. Don't forget to normalize.

   b) Find its (unnormalized) wavefunction $\psi(x)$ for real $\alpha$ in the position basis.

   c) Show that $\psi(x) = \phi(x - \alpha\sqrt{2})$, where $\phi(x)$ is the wavefunction of the superposition $c_0 |0\rangle + c_1 |1\rangle$ for some values of $c_0$ and $c_1$. Find these values.

Problem 2 (50 pts). Alice and Bob share two photons in state $|\Psi\rangle = (|HH\rangle - \sqrt{2}\,|HV\rangle - |VV\rangle)/2$ .

   a) Write the density matrix of this state.

   b) Alice measures the polarization of her photon in the canonical basis. What is the probability of each outcome and what state will be prepared at Bob's station in each case?

   c) Write the density matrix $\hat{\rho}_b$ of Bob's photon if he has no communication with Alice.

   d) Alice is using a polarization detector that measures in a canonical basis, but displays the correct result with probability 3/4, and the incorrect result with probability 1/4. What is the probability of each outcome and what state will be prepared at Bob's station in each case?

   e) Find the POVM of that detector.

All answers in Problem 2 should be presented in the matrix form, in the canonical basis.
**Hint:** It may be easier *not* to use the partial trace formalism.