University of Calgary
Fall semester 2013

PHYS 615: Advanced Quantum Mechanics I

Homework assignment 2

Due Monday October 06, 2013

Problem 2.1. Find the expectation value and uncertainty of operator $\hat{\sigma}_x \otimes \sigma_y$ in state $|\Psi\rangle = (|HH\rangle + 2|HV\rangle - 3 - |VV\rangle)/\sqrt{14}$.

Problem 2.2. Alice and Bob share the state $|\Psi\rangle = (|HH\rangle + 2|HV\rangle - 3|VV\rangle)/\sqrt{14}$.

a) Alice measures the state in the canonical basis. What is the probability of each outcome and what state will be prepared at Bob's station in each case?

b) Suppose Bob does not know Alice's result. Based on part (a), give a verbal description of the state of the photon at Bob's station and, based on that description, write the density matrix in the canonical basis.

c) Repeat parts (a) and (b) for Alice's measurement in the $|\pm 45°\rangle$ basis. Verify that Bob's density matrix in the canonical basis is the same.

d) Determine Bob's density matrix using the partial trace and check for consistency with the previous results.

Problem 2.3. What is the density matrix (in the canonical basis) of an ensemble of linearly polarized states with the polarization angle evenly distributed between 0 and $\pi/2$?

Problem 2.4. Entangled state $|\Psi\rangle = (|HV\rangle - 3|VH\rangle)/\sqrt{10}$ is used in the teleportation protocol. Find the probability for each of Alice's Bell basis measurement results and the corresponding state emerging in Bob's channel. Show that, for teleporting a linearly polarized state $|\theta\rangle$ for $\theta \ll 1$, this procedure can be used to "amplify" the vertical component of the input.

Problem 2.5. Consider the ensemble of photons that is

- in state $(3i|H\rangle + 4|V\rangle)/5$ with probability 1/2;
- in state $(12|H\rangle - 5|V\rangle)/13$ with probability 1/4;
- in state $|+45°\rangle$ with probability 1/4.

a) This ensemble is measured in the circular basis. Find the probabilities of each result using the verbal description above and using the density matrix formalism.

b) Present the ensemble as a mixture of two orthogonal states.