University of Calgary
Fall semester 2013

PHYS 615: Advanced Quantum Mechanics I

Homework assignment 3

Due Monday October 20, 2013

Problem 2.1. Consider two-mode optical state

$$|\psi\rangle = \sum_{k,m=0}^{\infty} \psi_{km} |k\rangle_1 \otimes |m\rangle_2 \,,$$

where subscripts 1 and 2 denote the modes (e.g. state $|1\rangle_1 \otimes |0\rangle_2$ means there is one photon in mode 1 and vacuum in mode 2).

a) Mode 2 is discarded. What is the density operator of the state in mode 1?

b) Mode 2 is subjected to measurement by a non-discriminating single-photon detector with quantum efficiency η. What is the density operator of the state in mode 1 in the event of a "click"?

c) Repeat (b) for the case when the initial state is not pure, but described by density matrix

$$\hat{\rho} = \sum_{k,l,m,n=0}^{\infty} \rho_{klmn} |k\rangle\langle l|_1 \otimes |m\rangle\langle n|_2 \,.$$

Problem 2.2. The beam of a pulsed laser of power $P = 1$ W, pulse repetition rate $f = 1$ MHz, wavelength $\lambda = 1064$ nm propagates through an attenuator with transmissivity $T = 10^{-13}$.

a) Find the amplitude α of the resulting coherent state associated with each of the pulses.

b) After attenuation, the beam is incident on a non-discriminating photon counter of quantum efficiency $\eta = 0.5$.[1] Find the mean count event rate.

Problem 2.3. Use a mathematical software package to calculate the Wigner function of the states below. Also the probability densities $\mathrm{pr}(X)$ and $\tilde{\mathrm{pr}}(P)$ for the position and momentum from the states' wavefunctions. Verify that these probability densities are obtained when the Wigner function is integrated over the other quadrature. Plot both probability densities and the Wigner function. If you have no 3D graphics software, plot relevant cross-sections of the Wigner function to show their shapes.

a) vacuum state;

b) coherent state with $\alpha = 2$;

c) the single-photon state;

d) the four-photon state;

e) $(|0\rangle + |1\rangle)/\sqrt{2}$;

f) $(|0\rangle + i\,|1\rangle)/\sqrt{2}$;

g) state with the density matrix $(|0\rangle\langle 0| + |1\rangle\langle 1|)/2$;

h) squeezed vacuum state $\psi_r(X) = \frac{1}{\pi^{1/4}\sqrt{r}} e^{-x^2/2r^2}$; plot for $r = 2$;

[1] "Non-discriminating" means that the "click" provides no information about the pulse energy: it will be the same whether the pulse contains 1 or 100 photons. The POVM of a non-discriminating detector has been reviewed in class.

i) Schrödinger cat state $|\alpha\rangle - |-\alpha\rangle$ with real $\alpha = 3$ (neglect normalization); plot for $\alpha = 3$.

j) state with the density matrix $(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)/2$; plot for $\alpha = 3$.

Problem 2.4. The Bell inequality is tested with the mixture of Bell states

$$\hat{\rho} = \eta \left|\Psi^-\right\rangle\left\langle\Psi^-\right| + (1-\eta) \left|\Psi^+\right\rangle\left\langle\Psi^+\right| ,$$

where $|\Psi^\pm\rangle = (|HV\rangle \pm |VH\rangle)/\sqrt{2}$. For which values of η will the Bell inequality be violated?