University of Calgary
Fall semester 2013

PHYS 615: Advanced Quantum Mechanics I

Homework assignment 4

Due Monday November 04, 2013

Problem 4.1. Alice and Bob share two photons in a polarization state whose matrix in the canonical basis is

$$\hat{\rho}_{ab} = \frac{1}{18}\begin{pmatrix} 3 & 1 & -2 & i \\ 1 & 1 & 2i & -3 \\ -2 & -2i & 4 & 0 \\ -i & -3 & 0 & 10 \end{pmatrix}$$

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a) Write the density matrix $\hat{\rho}_b$ of Bob's photon if he has no communication with Alice.

b) Alice measures the polarization of her photon in the canonical basis. What is the probability of each outcome and what state will be prepared at Bob's station in each case?

c) Find the POVM of the polarization detector that measures in a canonical basis, but may yield an incorret result:

- if the input state is $|H\rangle$, it will display H with probability 3/4, V with probability 1/4;
- if the input state is $|V\rangle$, it will display V with probability 2/3, H with probability 1/3.

d) Alice measures her photon using that detector. What is the probability of each outcome and what state will be prepared at Bob's station in each case?

All answers in this problem should be presented in the matrix form, in the canonical basis.

Problem 4.2. For the displacement operator $\hat{D}(X_0, 0)$:

a) Find $\hat{D}^\dagger(X_0,0)\hat{a}\hat{D}(X_0,0)$ and $\hat{D}^\dagger(X_0,0)\hat{a}^\dagger\hat{D}(X_0,0)$;

b) Find $[a, \hat{D}(X_0,0)]$ and $[a^\dagger, \hat{D}(X_0,0)]$;

c) Find the Fock decomposition of the *displaced single-photon state* $\hat{D}(X_0,0)\,|1\rangle$ and the photon number distribution associated with this state.

Problem 4.3. Consider the vacuum state evolving under Hamiltonian $\hat{H} = \xi\left[\hat{a}^2 + (\hat{a}^\dagger)^2\right]/2$, with real and positive ξ, for time t_0.

a) Find the mean and variance of the general quadrature $X_\theta = X\cos\theta + P\sin\theta$ for arbitrary angle θ.

b) Which angle corresponds to the highest squeezing?

c) What is the corresponding quadrature variance?

Problem 4.4. As derived in class, the Wigner function of any state $|\psi\rangle$ at the phase-space origin is $W_{|\psi\rangle}(0,0) = \left\langle\psi\middle|\,\hat{\Pi}\,\middle|\psi\right\rangle/\pi$, where $\hat{\Pi}$ is the parity operator. Using the fact that $W_{\hat{D}(X_0,P_0)|\psi\rangle} = W_{|\psi\rangle}(X - X_0, P - P_0)$, the parity operator can be used to calculate the state's Wigner function at any point of the phase space. Apply this method to calculate the Wigner function $W(X,0)$ of the vacuum and single-photon states using the parity operator representation

a) in the position basis;

b) in the number basis.

Do not use any software to solve this problem.